OCR
Oxford Cambridge and RSA

Tuesday 26 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the barcodes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 A "rugby sevens" team has to be chosen from 9 rugby players. The players have differing physiques and skills, which fit them for different positions, as shown in the table.

player	A	B	C	D	E	F	G	H	I
1 – tight head prop	✓	✓			✓				
2 – hooker		✓						✓	
3 – loose head prop	✓	✓			✓				
4 – scrum half			✓	✓			✓		
5 – fly half			✓				✓		
6 – centre			✓			✓		✓	✓
7 – wing			✓						✓

(i) Represent this information in a bipartite graph. **[2]**

The team manager starts to pick a team by choosing A to be tight head prop, B to be loose head prop, C to be wing, G to be scrum half and H to be centre.

(ii) Show this incomplete matching as a bipartite graph, and apply the matching algorithm to produce a complete matching. **[7]**

The team manager holds training sessions during which he assesses each player on a scale from 1 to 5 for each potential position, with 5 being the top score and 1 the lowest score. These scores are shown below.

player	A	B	C	D	E	F	G	H	I
1 – tight head prop	4	4			3				
2 – hooker		3						2	
3 – loose head prop	4	4			3				
4 – scrum half			5	4			4		
5 – fly half			5				4		
6 – centre			5			4		3	5
7 – wing			5						4

(iii) Formulate as a linear programming problem the problem of choosing the best team. **[5]**

(iv) Run your LP and interpret the solution. **[4]**

2 A mathematical model of an epidemic has the following features:

A Individuals are either susceptible, infective or recovered.

B A susceptible individual can be infected when in contact with an infective individual. The susceptible then becomes an infective.

C Infectives can recover. Recovered individuals are no longer infective or susceptible.

A simulation follows the following additional rules:

D In any hour at most one susceptible individual can become infective, and at most one infective individual can recover.

E The probability of an infection taking place during an hour is $\lambda \times$ the number of susceptible individuals at the beginning of the hour $\times$ the number of infective individuals at the beginning of the hour, where λ is a constant.

F The probability of a recovery taking place during an hour is $\beta \times$ the number of infective individuals at the beginning of the hour, where β is a constant.

Let the number of infective individuals at time t (hours) be $f(t)$ and the number of susceptible individuals at time t (hours) be $s(t)$. So, in the simulation, if a random number between 0 and 1 is less than $\lambda \times f(t) \times s(t)$, then there is a new infection in the hour beginning at time t hours.

The number of recovered individuals is modelled in a similar way, but following the rule given in F.

Ten people share a hut in a remote area, cut off by bad weather. Two are infective and the other eight are susceptible.

(i) Build an Excel simulation model for the hour-by-hour progress of the epidemic, allowing the input of specific values for λ and β. **[8]**

(ii) Run your simulation with $\lambda = 0.04$ and $\beta = 0.2$ until $f(t) = 0$, noting the time at which this happens and the value of $s(t)$ at that time. **[2]**

(iii) Repeat part **(ii)** 19 more times. Summarise and interpret your 20 results. **[5]**

(iv) Compare and contrast what happens if a medicine is available which reduces λ to 0.01 and increases β to 0.3. **[2]**

(v) Describe how to improve the simulation so that it more closely approximates to reality. **[1]**

3 Soil temperatures are measured at 10 cm beneath the surface at a number of locations at an agricultural research institute. It is suggested that the daily mean soil temperature across these sites can be modelled by a recurrence relation of the form $T_n = aT_{n-1} + bT_{n-2} + c$, where T_n is the mean soil temperature at the institute on day n.

(i) Solve the recurrence relation in the case $a = 0.8$, $b = 0.2$ and $c = 0$, i.e. find an expression in terms of n which will give the value of T_n. Start with $T_1 = 7$ and $T_2 = 8$. **[8]**

(ii) Construct a spreadsheet to show that your formula is correct for the first 20 terms. **[3]**

(iii) Give the exact value to which the model temperature converges. **[1]**

The "c" term allows for a daily increase in soil temperature if the weather is warm, and a daily decrease if the weather is cold.

(iv) Use your spreadsheet from part **(ii)** with $a = 0.8$ and $b = 0.2$, to investigate what happens to the modelled soil temperature if c is constant at a value of 0.1 over a period of 20 days. Describe what happens to the temperature differences. **[2]**

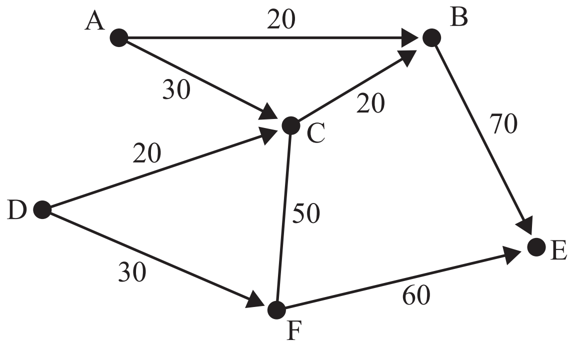
(v) Adapt your answer to part **(i)** to give a formula in terms of n which will give the value of T_n when $c = 0.1$ and when n is greater than 15, correct to the accuracy of your spreadsheet. **[1]**

(vi) Find values of a, b, and c which lead to modelled values which are close to the following observed values: **[3]**

n	1	2	3	4	5	6	7	8	9	10
T_n	7	8	7.8	8.08	8.17	8.33	8.47	8.61	8.75	8.90

n	11	12	13	14	15	16	17	18	19	20
T_n	9.04	9.18	9.33	9.47	9.61	9.76	9.90	10.04	10.18	10.33

4 A water supply network is shown in the diagram. The arcs represent pipes and the weights on the arcs represent pipe capacities. The pipe connecting C and F is undirected.



Pumps at A and D deliver water into the network. Pump A has capacity 40 and pump D has capacity 40. Water is extracted at B and E. B has a demand of 50 and E has a demand of 30.

(i) Add a super source and a super sink with associated arcs to model this situation. **[2]**

(ii) Find a set of feasible flows which give a total flow of 70 through the network, and prove that this is a maximal flow. Say which demand cannot be satisfied. **[3]**

(iii) Construct a linear program to model flows through the network. **[5]**

(iv) Run your program, interpret your results, and check that the maximal total flow is as per your answer to part **(ii)**. **[3]**

(v) Change your program to model the effect of decreasing the demand at B to 30 and increasing the demand at E to 50, and report on the result. **[2]**

New pumps are installed at A and D, each with capacity 50. Demands at B and E are now each 50.

(vi) Give 3 separate modifications to individual pipes, any one of which will allow the demands to be met. In each case give a set of feasible flows which satisfy the demands. **[3]**

END OF QUESTION PAPER

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