OCR
Oxford Cambridge and RSA

Tuesday 19 June 2018 – Afternoon

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The function $f(x)$ has the values shown in the table, correct to six decimal places.

x	1	1.125	1.25	1.5	2
$f(x)$	2.718282	2.888277	3.058835	3.403298	4.113250

Use the forward difference method to obtain four estimates of the gradient of $f(x)$ at $x = 1$.

Without doing any further calculation, state the value of $f'(1)$ to the accuracy that appears justified. **[6]**

2 **(i)** Write down the value of $2^{10} \times 2^{-10}$ and use your calculator to evaluate $2^{10} \div 2^{-10}$. **[2]**

My calculator displays the value of 2^{-10} as 0.000 976 562.

(ii) Calculate $2^{10} \times 0.000\,976\,562$ and $2^{10} \div 0.000\,976\,562$. **[2]**

(iii) Explain why your answers to part **(ii)** are different to your answers to part **(i)**. **[1]**

When I use my calculator to evaluate $2^{10} + 2^{-10}$ the value 1 024.000 977 is displayed. I obtain the same value when I enter $2^{10} + 0.000\,976\,562$.

(iv) Explain why there is no discrepancy between the values in this case. **[1]**

(v) Calculate the relative error in approximating 2^{-10} as 0.000 976 562. **Without** further calculation explain why this is the same as the relative error in the multiplication in part **(ii)**. **[2]**

3 The value of $\int_0^1 1.5^x \, dx$ is to be found.

(i) Use the mid-point rule with $h = 1$ and the trapezium rule with $h = 1$ to estimate the value of $\int_0^1 1.5^x \, dx$. **[3]**

(ii) Calculate a second estimate of the integral using the trapezium rule. **[1]**

(iii) **Without** doing any further calculation, state the value of $\int_0^1 1.5^x \, dx$ as accurately as you can. **[1]**

(iv) The mid-point rule estimate of this integral with $h = 0.5$ is 1.231 042 46, correct to eight decimal places. Use this and the estimates found in parts **(i)** and **(ii)** to obtain two Simpson's Rule estimates of $\int_0^1 1.5^x \, dx$ and hence write down the value of the integral to the accuracy that appears justified. **[3]**

4 The equation $x^2 + 10\,000x + 1 = 0$ has two roots, α and β, where $\beta > \alpha$.

(i) By completing the square, show that

$$\beta = \sqrt{24\,999\,999} - 5\,000. \qquad (*)$$

[2]

(ii) Show also that

$$\beta = \frac{-10\,000 + \sqrt{99\,999\,996}}{2}. \qquad (**)$$

[1]

A scientific calculator evaluates (*) as $-0.000\,100\,04$ and (**) as $-0.000\,100\,1$.

(iii) Explain why both of these values are likely to be inaccurate. **[1]**

(iv) Show that

$$\beta = -\frac{2}{10\,000 + \sqrt{99\,999\,996}}$$

and hence find an improved approximation for β. **[2]**

5 The equation $x - \log_{10} x - 2 = 0$ has a root in the interval $0 < x < 1$.

(i) Use one application of the secant method with $x_0 = 0.5$ and $x_1 = 0.2$ to find an approximation to this root. Explain why the method would fail to find a more accurate approximation to the root in this case. **[3]**

(ii) Use the secant method with $x_0 = 0.01$ and $x_1 = 0.02$ to obtain the root correct to five decimal places. **[3]**

(iii) State two reasons why the secant method may be preferred to the method of false position. **[2]**

Section B (36 marks)

6 The function $f(x)$ has the values shown in the table.

x	0	1	2	3
$f(x)$	-2	1	14	85

(i) Construct a table of differences as far as the third difference. **[3]**

(ii) Construct the Newton interpolating polynomial of degree 3, giving your answer in the form $ax^3 + bx^2 + cx + d$. **[5]**

(iii) Use your answer to **(ii)** to find

(A) an estimate of the gradient of $f(x)$ at $x = 2.5$, **[3]**

(B) an estimate of $\int_0^3 f(x)\,dx$. **[3]**

(iv) It is subsequently found that $f(4) = 358$. Determine whether this new information casts doubt on the reliability of your answers to parts **(iii) (A)** and **(B)**. **[4]**

7 **(i)** Show that the equation $x^5 - 6x + 4 = 0$ has a root, α, such that $-2 < \alpha < -1$. **[2]**

(ii) Show numerically that using the Newton-Raphson method with $x_0 = -1$ fails to find α. **[3]**

(iii) Use the iteration $x_{r+1} = \sqrt[5]{6x_r - 4}$ with $x_0 = -1$ to find α correct to 6 decimal places. **[3]**

There is another root of the equation, β, such that $0 < \beta < 1$.

The following iteration, with $x_0 = 0$, was used to find β.

$$x_{r+1} = \frac{x_r^5 + 4}{6} \qquad (*)$$

The results are shown in the table below.

The difference between successive iterates is shown in the third column, and the ratio of these differences is shown in the fourth column. The values in the fourth column are given to two decimal places.

r	x_r	$x_{r+1} - x_r$	$\dfrac{x_{r+2} - x_{r+1}}{x_{r+1} - x_r}$
0	0	0.666 667	0.03
1	0.666 667	0.021 948	0.18
2	0.688 615	0.003 858	0.19
3	0.692 473	0.000 731	0.19
4	0.693 204	0.000 141	0.19
5	0.693 345	0.000 027	0.19
6	0.693 372	0.000 005	0.20
7	0.693 377	0.000 001	–
8	0.693 378	0	
9	0.693 378		

(iv) What do the values in the fourth column tell you about the convergence of the iteration (*) with $x_0 = 0$? Justify your answer. **[2]**

(v) Use the Newton-Raphson method with $x_0 = 0$ to find β correct to 8 decimal places, showing the iterates in the table in the Printed Answer Book. Hence complete the table. **[6]**

(vi) What do your values in the fourth column tell you about the convergence of this iteration? Justify your answer. **[2]**

END OF QUESTION PAPER

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