OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Mark Scheme

1	(i)		Find a common point. e.g. set $z=0$ then solve $x+2y=11,\ 2x-y=7$ simultaneously. $\Rightarrow (5, 3, 0)$ Find the vector product $\Rightarrow n_1=\begin{pmatrix}1\\2\\-2\end{pmatrix},\quad n_2=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $\Rightarrow n_1\times n_2=\begin{pmatrix}0\\-5\\-5\end{pmatrix}\equiv\begin{pmatrix}0\\1\\1\end{pmatrix}$ $\Rightarrow \frac{x-5}{0}=\frac{y-3}{1}=\frac{z}{1}$ **oe** e.g. $x=5,\ y=z+3$ *Alternative scheme* **M1** Eliminating one variable **A1** For obtaining one of $x=5,\ y-z=3$ **M1** Eliminating another, or substituting **A1** For obtaining the other equation **A1** For the answer, as before	**M1** **A1** **M1** **A1** **A1**	Find a common point Evaluation of vector product **or** finding a second point and using it to find direction of *L* *Do not accept vector or parametric form*
				[5]	
	(ii)		$d_1=\left\lvert\frac{2a-1+1-7}{\sqrt{1^2+2^2+1^2}}\right\rvert=\left\lvert\frac{2a-7}{\sqrt{6}}\right\rvert$ $d_2=\left\lvert\frac{a+2-2-11}{\sqrt{1^2+2^2+2^2}}\right\rvert=\left\lvert\frac{a-11}{\sqrt{9}}\right\rvert=\left\lvert\frac{a-11}{3}\right\rvert$ $\Rightarrow \frac{2a-7}{\sqrt{6}}=\frac{a-11}{3}\Rightarrow a=\frac{21-11\sqrt{6}}{6-\sqrt{6}}=2-\frac{3}{2}\sqrt{6}$ **Or** $\frac{2a-7}{\sqrt{6}}=\frac{11-a}{3}\Rightarrow a=2+\frac{3}{2}\sqrt{6}$	**M1** **A1** **M1** **A1**	Using formula for distance *(one sufficient; modulus not required; M0 if neither 11 nor 7 used)* Both distances correct *(modulus not required)* Finding **two** equations (or squaring) Accept (art) $-1.67,\ 5.67$ *or* $\frac{8\pm\sqrt{216}}{4}$ *etc*
				[4]	

	(iii)		$1+4-14\neq 11$ but $2-2+7=7$ $1+0+10=11$ but $2-0-5\neq 7$	B1	www Needs evidence of substitution
			BC intersects *Q* at B only; and *P* at C only. B and C are not on *L* so BC does not intersect *L* BC is not parallel to *Q* (or *P*), so BC is not parallel to *L*	B1	Convincingly explain why BC does not intersect *L* **and** is not parallel to *L*
				[2]	
	(iv)		$\mathbf{d}=\text{CB}=\begin{pmatrix}1\\2\\7\end{pmatrix}-\begin{pmatrix}1\\0\\-5\end{pmatrix}=\begin{pmatrix}0\\2\\12\end{pmatrix}$, $\mathbf{e}=\begin{pmatrix}0\\1\\1\end{pmatrix}$	B1	For $\begin{pmatrix}0\\2\\12\end{pmatrix}$
			$\mathbf{d}\times\mathbf{e}=\begin{pmatrix}0\\2\\12\end{pmatrix}\times\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}-10\\0\\0\end{pmatrix}$, $\lvert\mathbf{d}\times\mathbf{e}\rvert=10$	M1 A1 ft	Vector product of directions Evaluation of vector product
			$\mathbf{a}-\mathbf{b}=\begin{pmatrix}4\\1\\-7\end{pmatrix}\Rightarrow(\mathbf{a}-\mathbf{b}).(\mathbf{d}\times\mathbf{e})$ $\left\lvert\dfrac{(\mathbf{a}-\mathbf{b}).(\mathbf{d}\times\mathbf{e})}{\lvert\mathbf{d}\times\mathbf{e}\rvert}\right\rvert=4$	M1 A1	Using correct formula to evaluate the distance cao
				[5]	

	(v)		$\mathbf{n}=\begin{pmatrix}2\\-1\\1\end{pmatrix}\Rightarrow$ CE has equation (CE is perpendicular to *Q*)	M1	*Alternative scheme* **M1** Finding distance from C to *Q* **A1** Distance is $\frac{5}{3}\sqrt{6}$ **A1** Length of CE is $\frac{10}{3}\sqrt{6}$ **A1** $\text{CE}=(\pm)\frac{10}{3}\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **or** E correct
			$\mathbf{r}=\begin{pmatrix}1\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}1+2\lambda\\-\lambda\\-5+\lambda\end{pmatrix}$ (Equation of CE)	A1	
			Meets *Q* when $2+4\lambda+\lambda-5+\lambda=7$ (Equation for λ) $\Rightarrow 6\lambda=10\Rightarrow\lambda=\frac{5}{3}$	A1	
			E corresponds to $\lambda=\frac{10}{3}$ So E has coordinates $\left(\frac{23}{3},-\frac{10}{3},-\frac{5}{3}\right)$	A1	
			Volume of tetrahedron $=\frac{1}{6}\left\|OE.(OB\times OC)\right\|$	M1	Appropriate scalar triple product stated (any 3 non-coplanar edges) (1/6 not required)
			$=\frac{1}{6}\left\|\begin{pmatrix}\frac{23}{3}\\\frac{-10}{3}\\\frac{-5}{3}\end{pmatrix}.\left(\begin{pmatrix}1\\2\\7\end{pmatrix}\times\begin{pmatrix}1\\0\\-5\end{pmatrix}\right)\right\|=\frac{1}{6}\left\|\begin{pmatrix}\frac{23}{3}\\\frac{-10}{3}\\\frac{-5}{3}\end{pmatrix}.\begin{pmatrix}-10\\12\\-2\end{pmatrix}\right\|$	A1 ft	Correct vectors substituted into correct formula (including 1/6)
				A1 ft	Evaluation of vector product
			$=\frac{1}{6}\left\|\left(-\frac{230}{3}-\frac{120}{3}+\frac{10}{3}\right)\right\|=\frac{340}{18}$ $\Rightarrow$ Volume of solid $=\frac{170}{9}$	A1	cao Accept 18.9
				[8]	

2	(i)		$\frac{\partial z}{\partial x}=1+8xy,\quad \frac{\partial z}{\partial y}=4x^2-4y$	B1B1	
			At P (1, 1, 5), $\frac{\partial z}{\partial x}=9,\ \frac{\partial z}{\partial y}=0$	B1	
			Tangent plane at is $z-c=\left.\frac{\partial z}{\partial x}\right\vert_{(a,b)}(x-a)+\left.\frac{\partial z}{\partial y}\right\vert_{(a,b)}(y-b)$	M1	**or** $z=9x+d$ **or** $9x-z=d$
			$\Rightarrow z-5=9(x-1)+0(x-1)\Rightarrow z-5=9(x-1)\Rightarrow z=9x-4$	A1	**AG**
				[5]	
	(ii)		For tangent plane to be $z=9x+k$, $\frac{\partial z}{\partial x}=9$ and $\frac{\partial z}{\partial y}=0$	M1	
			$\frac{\partial z}{\partial y}=4x^2-4y=0\Rightarrow y=x^2$,		
			$\frac{\partial z}{\partial x}=1+8xy\Rightarrow 1+8x^3=9\Rightarrow x=1$	A1	**or** $y=1$
			$\Rightarrow y=1\ \Rightarrow z=5$ So z = 9x + k only at (1, 1, 5) ⇒k = −4	A1	**AG** Must mention −4
				[3]	
	(iii)		$\delta z\approx\left.\frac{\partial z}{\partial x}\right\vert_P\delta x+\left.\frac{\partial z}{\partial y}\right\vert_P\delta y$ ($\delta z\approx 9\delta x+0\delta y$) $\Rightarrow b=9a+0\Rightarrow b=9a$	M1 A1 ft A1	*Alternative scheme* **M1** Substituting $x=y=1+a,\ z=5+b$ **A1** For $b=9a+10a^2+4a^3$ **A1** For $b\approx 9a$ since a is small
				[3]	

	(iv)		$\frac{\partial z}{\partial x}=1$ and $\frac{\partial z}{\partial y}=16$ $\Rightarrow 1+8xy=1 \Rightarrow xy=0 \Rightarrow x=0$ or $y=0$ $\Rightarrow 4x^2-4y=16 \Rightarrow x^2=y+4$ $x=0 \Rightarrow y=-4, z=-30$ i.e. $(0,-4,-30)$ $y=0 \Rightarrow x=\pm 2, z=0,4$ i.e. $(2,0,4)$ and $(-2,0,0)$	M1 A1 A1A1	Allow M1 for −1, −16 M0 for *λ*, 16*λ* (unless $\lambda=\pm 1$ seen later) **SC** If M1A0 then SC A1 for all *x* and *y* values correct
				[4]	
	(v)		$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0 \Rightarrow 1+8xy=0,\ \ 4x^2-4y=0$ $\Rightarrow y=x^2, 1+8xy=0 \Rightarrow x^3=-\frac{1}{8} \Rightarrow x=-\frac{1}{2}$ $\Rightarrow y=\frac{1}{4}, z=-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+2=\frac{13}{8}$ $x=-\frac{1}{2}$ is the only root of the equation $1+8x^3=0$ So only one stationary point. Set $x=-\frac{1}{2} \Rightarrow z=\frac{3}{2}+y-2y^2$ Set $y=\frac{1}{4} \Rightarrow z=x+x^2+\frac{15}{8}$ One section has a maximum and the other has a minimum meaning that the stationary point is a saddle point	M1 M1 A1 A1 A1 M1 A1 A1 A1	For $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0$ Obtaining an equation in *x* (or y) only AG For $x=-\frac{1}{2}$ (*or* $y=\frac{1}{4}$) AG For other two coordinates *(some evidence required for z)* AG *Must refer to this being the only solution (or the only stationary point)* Finding cross-sections *(one sufficient)* *(M0 for* $x=0$ *and* $y=0$ *used)* *Correct diagrams sufficient*
				[9]	

3	(a)		Formula for surface area = $\int_{x_1}^{x_2} 2\pi y \,\mathrm{d}s$	M1	Used with equation of a circle
			$\mathrm{d}s = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ *or* $\mathrm{d}s = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$	M1	Used
			$x^2+y^2=a^2 \Rightarrow y=\sqrt{a^2-x^2}$, *or* $x=a\cos\theta,\ y=a\sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x}{y}$ *or* $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-a\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta}=a\cos\theta$	B1	(in any form)
			$S=2\int_0^a 2\pi y\sqrt{1+\frac{x^2}{y^2}}\mathrm{d}x$ *or* $S=\int_0^\pi 2\pi a\sin\theta\sqrt{a^2\sin^2\theta+a^2\cos^2\theta}\,\mathrm{d}\theta$	A1	Correct integral expression for S including limits
			$=4\pi a\int_0^a \mathrm{d}x$ $\quad = 2\pi a^2\int_0^\pi \sin\theta\,\mathrm{d}\theta$	A1	For $y\,\mathrm{d}s = a\,\mathrm{d}x$ or $y\,\mathrm{d}s = a^2\sin\theta\,\mathrm{d}\theta$
			$=4\pi a^2$	A1	AG
				[6]	
	(b)	(i)	$y=0 \Rightarrow 3t^3=4t$	M1	
			$t\neq 0 \Rightarrow t=\pm\frac{2}{3}\sqrt{3}$	A1	Accept $\pm\frac{2}{\sqrt{3}}$ (A0 if $t=0$ included)
				[2]	
		(ii)	$x=6t^2 \Rightarrow \dot{x}=12t$	B1	
			$y=4t-3t^3 \Rightarrow \dot{y}=4-9t^2$	B1	
			$\dot{x}^2+\dot{y}^2=\left(4+9t^2\right)^2$	M1 A1	Substituting into $\dot{x}^2+\dot{y}^2$
			$\Rightarrow s=\int_0^{\frac{2}{3}\sqrt{3}}\left(4+9t^2\right)\mathrm{d}t=\left[4t+3t^3\right]_0^{\frac{2}{3}\sqrt{3}}=\frac{2}{3}\sqrt{3}(4+4)=\frac{16}{3}\sqrt{3}$	M1 A1	For $\int\sqrt{\dot{x}^2+\dot{y}^2}\,\mathrm{d}t$ (limits not required)
				[6]	

		(iii)	$\ddot{x}=12,\quad \ddot{y}=-18t\ (=-6)$	B1	
			$\rho=\dfrac{\left(\dot{x}^2+\dot{y}^2\right)^{3/2}}{\dot{x}\ddot{y}-\ddot{x}\dot{y}}=\dfrac{\left(4+9t^2\right)^3}{-216t^2-12\left(4-9t^2\right)}=\dfrac{\left(4+9t^2\right)^3}{-48-108t^2}$	M1 A1	Using formula for ρ or κ Correct substitution (algebraic or numerical)
			$=\dfrac{\left(4+9t^2\right)^3}{-12\left(4+9t^2\right)}=-\dfrac{1}{12}\left(4+9t^2\right)^2$		
			When $t=\dfrac{1}{3}$ $\rho=(-)\dfrac{25}{12}$	A1	
				M1	Used
			Centre is $\mathbf{c}=\mathbf{r}+\rho\hat{\mathbf{n}}$	B1	
			When $t=\dfrac{1}{3}$, $\mathbf{r}=\begin{pmatrix}\frac{2}{3}\\ \frac{11}{9}\end{pmatrix}$		Finding $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
				M1	
			$\tan\psi=-\dfrac{4-9t^2}{12t}=\dfrac{3}{4}$	A1	Correct unit normal
			$\Rightarrow \hat{\mathbf{n}}=(\pm)\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$	A1	
				A1	
			$\Rightarrow \mathbf{c}=\begin{pmatrix}\frac{2}{3}\\ \frac{11}{9}\end{pmatrix}-\dfrac{25}{12}\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}=\begin{pmatrix}\frac{23}{12}\\ -\frac{4}{9}\end{pmatrix}$		
				[10]	

4	**(i)**		Order 2: 4,11, 14 Order 4: 2, 7, 8, 13 1 | 2 | 4 | 7 | 8 | 11 | 13 | 14 1 | 4 | 2 | 4 | 4 | 2 | 4 | 2	**B4**	B3 one error, B2 2 errors, B1 3 errors
				[4]	
	(ii)	**(*A*)**	1, 2, 4, 8	**B2**	B1 for one omission
				[2]	
		(*B*)	Order 2: {1, 4}, {1, 11},{1, 14} Order 4: {1, 2, 4, 8},{1, 4, 7, 13}	**B4**	B1 one correct of order 2 B1 one correct of order 4 B1 any 4 correct B1 all 5 correct with no extras *Ignore {1} if included*
				[4]	
	(iii)		(A) Yes. Closed 0 is the identity element Each element has an inverse 1,7 2,6 and 3,5 are inverse pairs, 4 is self inverse Not isomorphic e.g. G has 4 self inverse elements, this has 2 (B) No e.g. 3 × 3 = 0 so set not closed under this operation	 **B1** **B1** **B1** **B1** **B1** **B1** **B1** **B1**	 Giving inverses correctly (0 not needed) With reason Reason

			(C) Yes $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity All have inverses $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the inverse of $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ All other elements are self inverse. Not isomorphic as this one has 6 self-inverse elements	**B1** **B1** **B1** **B1** **B1** **B1**	(or vice-versa) With reason
				[14]	

5	**(i)**		Lights always change – they are never on for consecutive minutes. The run length is therefore 0 for each	**B1** **B1**	 Accept run length 0 or 1
				[2]	
	(ii)		Finding powers of P_1 0, 0.27, 0.162, 0.2091	**M1** **A1** **A1A1**	 *If M1A0 then SC A1 for three of 0, 0.27, 0.162, 0.2091 regardless of times given*
				[4]	
	(iii)		$0.162 < a < 0.2091$	**M1** **A1 ft**	For lower limit 0 or 0.162 and upper limit 0.27 or 0.2091
				[2]	
	(iv)		$\mathbf{P}_1^4 = \begin{pmatrix} \mathbf{0.2091} & 0.1976 & 0.201 & 0.1762 \\ 0.329 & \mathbf{0.3332} & 0.257 & 0.2492 \\ 0.2088 & 0.2054 & \mathbf{0.2529} & 0.256 \\ 0.2531 & 0.2638 & 0.2891 & \mathbf{0.3186} \end{pmatrix}$ probabilities for $t = 1$ are 0, 0.2, 0.3, 0.5 $\text{P}(\text{same at } t = 1 \text{ and } 5) = 0.2 \times 0.3332 + 0.3 \times 0.2529 + 0.5 \times 0.3186 = 0.302$ $\Rightarrow \text{P}(\text{different}) = 0.698$	**M1** **A1** **M1** **M1** **A1**	Using **diagonal** elements from P^4. Diagonal elements correct Using probabilities from 2nd minute Method for P(same) or P(different)
				[5]	
	(v)		$P_2 = \begin{matrix} & A & B & C & D \\ A & 0 & 0.5 & 0 & 0 \\ B & 1 & 0 & 0.5 & 0 \\ C & 0 & 0.5 & 0 & 1 \\ D & 0 & 0 & 0.5 & 0 \end{matrix}$	**B2**	B1 one error
				[2]	
	(vi)		**(A)** There are no absorbing states. **(B)** A and D are reflecting barriers	**B1** **B1B1**	 Maximum B1 if any extras
				[3]	

	(vii)		Algebraically: $\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}$ *Alternative scheme* **M1** Finding large even **and** large odd powers **M1** Averaging over even and odd powers **A4** Answers, which must be given as fractions	**M1** **M1** **A4**	Using column vector Creating and solving equations *(needs to use* $a+b+c+d=1$*)* A1 for each
				[6]	

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