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Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4753/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4753/01 series overview

This paper proved to be similar in standard to those of recent years, and we saw many excellent scripts. All candidates had sufficient time to complete all the questions. Very few scripts scored fewer than half marks, and a significant percentage scored full marks, suggesting that all the questions were accessible for well-prepared candidates.

It is important in 'show' questions that all the necessary steps are included: sometimes, for example in questions 8(i) and 8(iv), working was missing which was penalised.

Most successful questions	*Least successful questions*
• Q1 (implicit differentiation) • Q3 (exponential decay) • Q4 (product rule for differentiation)	• Q7 (proof) • Q8(iii) integration by substitution • Q9(iv) integration by parts/substitution

Key

AfL Guidance to offer for future teaching and learning practice.

Misconception

Section A overview

Candidates scored heavily on these questions, which proved to be accessible to all but a few. Topics such as linear inequalities (Q5), inverse trigonometric functions (Q6) and proof (Q6) were, in general, better answered than in recent years.

Question 1(i)

1 A point P moves round the curve with equation $3x^2 + 4y^2 = 4$. At time t, P is at the point (x, y), as shown in Fig. 1.

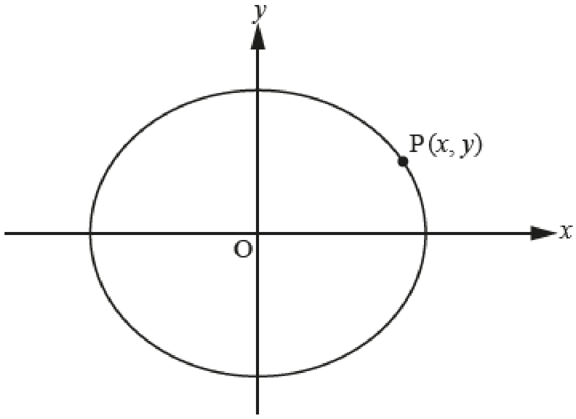

Fig. 1

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[2]**

Virtually all candidates scored 2 marks. Occasional attempts to solve for *y* then differentiate were usually unsuccessful.

Question 1(ii)

(ii) When P is at the point on the curve with x-coordinate 1 and positive y-coordinate, $\frac{dx}{dt} = 4$.

Find $\frac{dy}{dt}$ at this point. **[4]**

All gained M1 for the chain rule. A few neglected to find *y* when *x* = 1 by substituting.

Question 2

2 The three functions $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$$f(x) = \frac{x}{1-2x^2}, \quad g(x) = 1 + \sin 2x \quad \text{and} \quad h(x) = 3e^{-2x^2}.$$

In the table in the Answer Book, write Yes or No in each space to indicate whether the function is odd, whether it is even, and whether it is periodic. If a function is periodic, state its period. **[4]**

'Periodic' was less well understood than 'odd' and 'even'. Many also stated an incorrect period. A fairly common error was to state that 1 + sin 2*x* is odd (presumably because sin 2*x* is odd). Candidates were required to tick and cross every box – leaving a box blank could mean that they were undecided. Some candidates might have intended a blank entry to mean 'no'; this was not credited marks.

Question 3(i)

3 The mass of a radioactive material decreases exponentially. Its *half-life* is the time required for the mass of the material to reduce to half its initial value. The half-life of plutonium 241 is 14.4 years.

(i) Write down the percentage of the initial mass of plutonium 241 remaining after 28.8 years. **[1]**

A few gave the answer zero here, but most responses were correct.

Question 3(ii)

(ii) The mass M grams of plutonium 241 at time t years is given by the equation

$$M = M_0 e^{-kt},$$

where M_0 grams is the initial mass and k is a constant. Find k, giving your answer correct to two significant figures. **[3]**

Some used $\frac{1}{4} = e^{-28.8k}$ instead of $\frac{1}{2} = e^{-14.4k}$ here, which was of course perfectly acceptable. The logarithm work was generally very well done, and most candidates gained all three marks. Occasionally the answer was incorrectly rounded to 2 significant figures to 0.05.

Question 4

4 Fig. 4 shows part of the curve with equation $y = e^{-x} \sin 2x$.

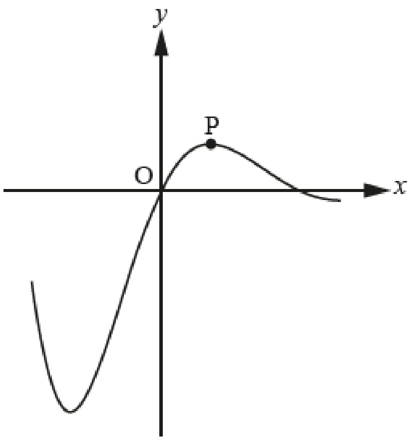

Fig. 4

Find the coordinates of the maximum point P. **[6]**

The product rule was well done here, and the subsequent trigonometric equation using $\frac{\sin}{\cos} = \tan$ was usually successful, although some expanded the sine and cosine of the double angle and then could progress no further.

Question 5(i)

5 (i) On the same axes, sketch the graphs of $y = -|x+1|$ and $y = 2x$. **[3]**

Candidates who did not label the point (–1, 0) lost a mark. It was also required that the gradient of the $y = 2x$ line to be steeper than the lines of the modulus graph.

Question 5(ii)

(ii) Solve the equation $-|x+1| = 2x$. **[2]**

Many candidates correctly solved the equation to get $x = -\frac{1}{3}$ and 1, but did not see the point of the sketch in part (i), which rejects the $x = 1$ solution. A few candidates used the squaring method here, occasionally getting $-(x + 1)^2 = 4x^2$.

Another mistake seen in a few scripts was $|x + 1| = |x| + 1$.

Question 6(i)

6 The function $\mathrm{h}(x)$ is such that $\mathrm{h}(x) = \mathrm{fg}(x)$, where $\mathrm{f}(x) = 2x + \frac{1}{2}\pi$ for $x \in \mathbb{R}$ and $\mathrm{g}(x) = \arcsin x$ for $-1 \leqslant x \leqslant 1$.

(i) Find $\mathrm{h}\left(\frac{1}{2}\right)$, giving your answer as a multiple of π. **[2]**

Question 6(ii)

(ii) Find $\mathrm{h}^{-1}(x)$. **[4]**

Both parts of this question were very well answered, using $\arcsin\frac{1}{2} = \frac{\pi}{6}$, then inverting the equation to find the inverse function.

Question 7

7 Prove that $n^3 - 3n^2 + 2n$ is divisible by 6 for all positive integers n. **[5]**

This 'proof' question was more successfully done than others from recent papers. Most candidates factorised correctly and recognised the product of consecutive integers contained one with a factor of 2 and one with a factor of 3, and some minor faults in their reasoning were condoned (for example, some said one was even and another a multiple of 3; others said the 3 consecutive integers contained 'at least' one multiple of 3). Those who did not factorise, however, usually got nowhere, though it is possible to argue that the given expression is even by considering the parity of the individual terms.

Section B overview

Both Section B questions were generally quite well answered, though the longer, more extended parts, required more accurate and sustained work to get to the correct answers.

Some candidates are still approximating answers which are required to be exact.

Question 8(i)

8 Fig. 8 shows the curve with equation $y = \frac{2\sqrt{x}}{1+\sqrt{x}}$. The tangent to the curve at P (1, 1) intersects the y-axis at Q.

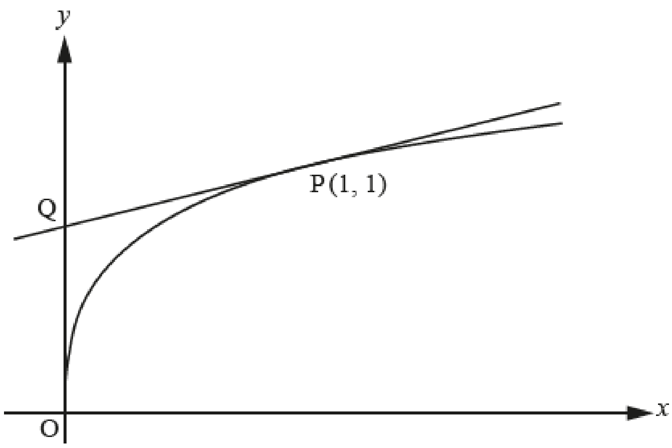

Fig. 8

(i) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{x}(1+\sqrt{x})^2}$.

Hence find the equation of PQ, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

This was a pretty straightforward application of the quotient rule, which was usually correctly done. However, many candidates did not show convincingly that their expression simplified to the given answer.

The equation of the tangent was then routine and well done, although a few left the coefficients as fractions.

Question 8(ii)

(ii) Show that the substitution $u = 1 + \sqrt{x}$ transforms $\int \frac{2\sqrt{x}}{1+\sqrt{x}} dx$ to $\int \frac{4(u-1)^2}{u} du$. **[3]**

Some candidates did not show enough work here to be convincing.

It is vital in integration by substitution that 'dx' and 'du' are shown as part of the integrand.

Question 8(iii)

(iii) Hence find the exact area of the region enclosed by the curve, the y-axis and the line PQ. **[8]**

This required more extended work. Most candidates calculated the area under the trapezium under PQ, though a minority used the integral of the line equation. When it came to integrating under the curve, most used the transformed integral and changed the limits, but then many did not spot the correct method here, namely expanding and dividing through by *u*. Others made errors with the expansion here. Nevertheless, many managed to navigate the difficulties to get the correct exact expression.

Question 9(i)

9 Fig. 9 shows the curves with equations $y = \ln x$ and $y = 2\ln(x-2)$ which intersect at Q. The curve $y = 2\ln(x-2)$ crosses the x-axis at P.

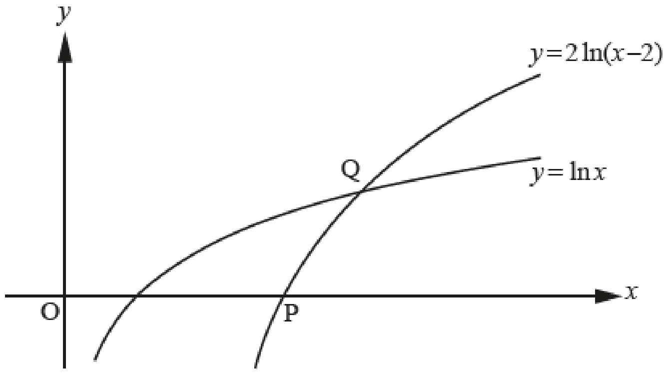

Fig. 9

(i) Describe a sequence of two transformations which maps the curve $y = \ln x$ onto the curve $y = 2\ln(x-2)$. **[3]**

The sequence of transformations was described correctly by most candidates. 'Shift' for translation was allowed, but not 'move'.

Question 9(ii)

(ii) Find the exact coordinates of P and Q. **[5]**

Most found P correctly, but there were a surprising number of incorrect derivations of the *x*-coordinate of Q, most common of which was:

$2\ln(x-2) = \ln x$, so $2(x-2) = x$.

Many of these incorrect methods seemed to lead fortuitously to $x = 4$ but gained no marks.

Question 9(iii)

(iii) Using integration by parts, show that $\int \ln x \, dx = x\ln x - x + c$, where c is an arbitrary constant. **[3]**

This standard application of integration by parts was well done, although candidates were required to resolve the $\frac{x}{x} = 1$ in the resulting integral part.

Question 9(iv)

(iv) Hence show that the area of the finite region enclosed by the curve $y = \ln x$, the curve $y = 2\ln(x-2)$ and the x-axis is $m\ln 2 + n$, where m and n are integers to be determined. **[7]**

Here, the difficulty for many candidates was in dealing with $\int 2\ln(x-2)\mathrm{d}x$. Perhaps influenced by the previous part, most candidates applied parts to this but struggled trying to integrate $\frac{x}{(x-2)}$. Only the most competent candidates recognised that substitution was a more appropriate method and saw this work through successfully to the correct final answer.

Candidates need to practice questions where they have to decide which integration method (by parts or by substitution) is most appropriate to use.

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