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Examiners' report

MATHEMATICS (MEI)

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4755/01 Summer 2018 series

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4755/01 series overview

The overall standard this year was high. Very few candidates appeared to be severely challenged by the questions and most were able to complete the examination in the time allowed.

On the whole the concepts of complex numbers, proof by induction and curve sketching were well understood, matrices less so. There was frequently shown to be a poor grasp of the rules of matrix multiplication.

Candidates do well when they automatically write correct expressions both legibly and with careful attention to signs and the need for brackets. Their sketches need to be made with care and full annotation. Calculator use is available but there is still a need to show full working or justification for results.

As usual, algebraic manipulation is not as good as it should be and candidates need to remember what happens to signs when brackets are inserted and negative numbers are involved, skills that should by this stage be well understood and used. Some answers were spoiled by arithmetical slips.

Section A overview

Questions 1, 4, 5 and 6 were on the whole the best answered in this section. The questions that caused candidates the most difficulty were 2, 3(ii) and 3(iii). Question 2 produced an interesting variety of solutions for the finding of *p*.

Question 1(i)

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 2k & -k \\ 0 & 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & -3 \\ -2 & 4 \end{pmatrix}$, where k is a constant.

(i) Find, in terms of k, the matrix **AB**. **[2]**

Nearly all candidates began well earning these two marks. There were mistakes involving signs. In working through the second row 3×2 and -3×-4 were seen. Rarely, the product **BA** was found instead of **AB**.

Question 1(ii)

(ii) Find the value of k for which matrix **AB** is singular. **[2]**

The vast majority of candidates understood what was required here and gained the 2 marks. Of the errors the most common was to go from e.g. $6k = -34$ to $k = \frac{17}{3}$, omitting the minus sign. A few candidates thought that the determinant should equal 1. Some confused the determinant with $\frac{1}{\text{determinant}}$, somehow (incorrectly) leading to the correct result.

Question 2

2 The quadratic equation $x^2 + px + q = 0$ has roots α and β, where

$$\alpha^2 + \beta^2 = -16,$$
$$\alpha - \beta = 6\text{j}.$$

By considering $(\alpha - \beta)^2$, find the value of $\alpha\beta$. Hence state the value of q and find the possible values of p. **[5]**

Most candidates earned the 3 marks for finding *αβ* and *q,* although some preferred not to state the value of *αβ* explicitly, rather ignoring the request.

Many were unable to find *p*.

The errors most often seen when finding *αβ* and *q* were:-

- squaring 6j to get –6
- expanding $(\alpha - \beta)^2$ as $\alpha^2 + \beta^2 - \alpha\beta$
- thinking that the product of the roots was –*q*
- writing $(\alpha - \beta)^2 = 0$ and rearranging to get $\alpha\beta = -8$.

Surprisingly many candidates ignored using the obvious expansion of $(\alpha + \beta)^2$ when attempting to find *p*. Errors seen were:-

- assuming that the roots occurred as complex conjugate pairs. This is not stated in the question (compare with question 5 where the coefficients are given to be real).
- misunderstanding the question wording "… possible values of *p*" by looking at the discriminant and starting with $p^2 - 4q > 0$ ....
- solving simultaneous equations by substituting $\alpha = 6\text{j} + \beta$ (or $\beta = 6\text{j} + \alpha$) in $\alpha^2 + \beta^2 = -16$ and ending up with a fourth-order equation. They could usually solve this to find values for α^2 (or β^2) but were unable to proceed. Only two of the four solutions led to the correct value of *αβ*.
- solving simultaneous equations by substituting $\alpha = \frac{10}{\beta}$ (or $\beta = \frac{10}{\alpha}$) in $\alpha - \beta = 6\text{j}$. They would solve the resulting equation using the formula but instead of writing (e.g.) $\alpha = 3\text{j} + 1$ and $3\text{j} - 1$ (which their formula indicated) they would write $\alpha = +3\text{j} + 1$ and $-3\text{j} - 1$ (presumably because they were so accustomed to seeing the complex solutions to quadratic equations as $a \pm b\text{j}$). This was seen many times.

Question 3(i)

3 **(i)** Sketch on an Argand diagram the set of points representing complex numbers z for which

$$|z - (3 + 3\text{j})| = 3. \qquad \textbf{[2]}$$

Most candidates earned both marks. Some candidates did not annotate their sketch and so lost a mark.

Question 3(ii)

(ii) Find the greatest possible value of $|z|$ for this set of points. **[2]**

This was reasonably well done. The most common error was for candidates to think that the greatest possible value of |*z*| occurred when *z* was at the point 6 + 3j and so $\sqrt{45}$ e was often seen. Some candidates also thought *z* was at 6 + 6j.

Question 3(iii)

(iii) Mark on your Argand diagram the particular point for which $\arg\left(z-(3+3\mathrm{j})\right)=\frac{2}{3}\pi$. Find this value of z in the form $a+\mathrm{j}b$. **[3]**

Marks were lost by candidates marking *z* but not indicating the angle $\frac{2}{3}\pi$ on their diagram. The main error was that candidates just used *z* as $3\left[\cos\left(\frac{2}{3}\pi\right)+\mathrm{j}\sin\left(\frac{2}{3}\pi\right)\right]$ without thinking where the point was. Candidates who found the real part of *z* to be $\frac{3}{2}$ should have shown explicitly that this came from $3+3\cos\left(\frac{2}{3}\pi\right)$, or equivalently $3-3\cos\left(\frac{1}{3}\pi\right)$.

Question 4(i)

4 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n} r(2+3r)=\tfrac{1}{2}n(n+1)(2n+3).$$ **[4]**

Almost all candidates earned 4 marks here, the odd error being in the algebraic manipulation. Some candidates multiplied out and factorised back rather than spotting the common factors and saving time.

Question 4(ii)

(ii) Hence find the value of n such that

$$\sum_{r=1}^{4n} r(2+3r)=198n(4n+1).$$ **[3]**

This question part was also done well. There were two main areas where candidates went astray.

- Some candidates substituted 4*n* in place of *n* only once i.e. simply multiplied the expression by 4 to give $2n(n+1)(2n+1) = 198n(4n+1)$.
- Some candidates solved correctly but gave either $n = 0$ or $n = -\frac{1}{4}$ (or both) as answers.

Quite a few candidates did not see that the (correct) equation could be factorised/cancelled down to give $8n+3 = 99$ and instead multiplied the expression out and rearranged. This led to a few errors in the algebraic manipulation.

Question 5

5 You are given that $z = 2 + 5\text{j}$ is a root of the cubic equation $2z^3 - 5z^2 + pz + q = 0$, where p and q are real constants. Find the values of p and q. **[6]**

This question was also well done, with many candidates earning full marks. There were three main methods used, as outlined in the mark scheme and the responses were split fairly equally between the three methods. Some candidates used a mixture of methods.

Those who evaluated $(2 + 5\text{j})^2$ and $(2 + 5\text{j})^3$ were the most prone to making errors in the algebraic manipulation, but almost all gained the 3 method marks.

Those who produced a quadratic factor probably fared best, if only slightly. The two chief sources of error were in the quadratic factor itself, where $z^2 - 4z - 21$, $z^2 - 4z + 9$ and $z^2 - 4z + 25$ were all seen.

The main errors using the sums and product of roots were either the manipulation (especially the sum of the pairs of products), or writing that the sum of the roots was $+\frac{5}{2}$, the sum of the product-pairs was $-\frac{p}{2}$, or that the product of the roots was $\frac{q}{2}$.

Question 6

6 Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} r2^r = 2[1 + (n-1)2^n]$. **[7]**

This question topic was encouragingly well answered this year. The majority of candidates were able to work through the inductive process correctly, following closely the argument which has always been advocated in the mark scheme and in these reports.

There are still careless omissions in presentation. Some begin "assume $n = k$" or "$n = k$". Often the $(k + 1)$th term is written as equal to the sum plus this last term. Where sigma appears the notation used often moves k into the general term instead of the limit of summation.

There are still candidates who try to finish off with their own wording, for example "... and so it is true for $n = k$ and for $n = k + 1$". The phrase "$n = k$ is true so $n = k + 1$ is true" also makes its regular appearance.

Clear handwriting is advised to ensure that indices do not become confused with coefficients e.g. 2^{k+1} becoming $2k + 1$ in the following line.

Another source of error came about when candidates multiplied out the whole expression. All the '2's and 'k's caused confusion thus some candidates ended up with $2k^k$ instead of $k2^k$, which was seen fairly often.

Section B overview

Question 7 was by far the most successfully answered question in this section. Question 8(i) required clear presentation which was not always given, and question 8(ii) caused many candidates some difficulty. Parts of question 9 were well answered. Matrix multiplication in question 9(ii) was a very weak area, with incorrect working often leading to an apparently correct result.

Question 7(i)

7 A curve has equation $y = \dfrac{2x^2 - 5x - 3}{x^2 + x - 2}$.

(i) Find the values of x for which $y = 0$. [2]

Nearly all candidates gained 2 marks here. Errors, such as factorising incorrectly, e.g. $(2x - 1)(x + 3)$ or equating the denominator to zero, were few and far between.

Question 7(ii)

(ii) Find the equations of the three asymptotes. [3]

Again, nearly all candidates gained full marks here. There were the usual errors, writing $x = 1, -2$ instead of $x = 1$ and $x = -2$, or writing $y \to 2$ instead of $y = 2$, and of course factorising as $(x + 1)(x - 2)$.

Question 7(iii)

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [2]

Candidates need to be aware of what constitutes mathematical justification for this type of question. Unsupported statements such as "when x is large the curve approaches the value 2 from below …." is insufficient. Using a graphical calculator the result can be seen, and the wording of the question is "determine whether…". It should be emphasised to candidates that they need to support their statements by for example choosing a large value of x, substituting this value into the expression and producing an answer before stating the result. Any work shown must be complete and thorough. Very able candidates lost marks here.

Question 7(iv)

(iv) Sketch the curve. [3]

Overall this question was answered well. Where a mark was lost it was usually through neglecting to annotate the points where the graph crossed the axis at $x = -\dfrac{1}{2}$ and at $y = \dfrac{3}{2}$. This latter point was not part of the previous requests and candidates need to remember that a sketch should show all salient points like this. Some annotations were extremely difficult to read after the scanning process, and on the x-axis '1' in particular had a tendency to disappear into the dashed line which showed the asymptote.

Question 7(v)

(v) Solve the inequality $\dfrac{2x^2-5x-3}{x^2+x-2} \geqslant 0$. **[3]**

Apart from candidates who had gone astray in parts (i) or (ii), the chief errors made were in the inequality signs. Often seen were $x \le -2$, $x > 3$, $-\dfrac{1}{2} < x < 1$ or $-\dfrac{1}{2} \le x \le 1$.

8 You are given that $\dfrac{1}{2r-1} - \dfrac{1}{2r+3} = \dfrac{4}{(2r-1)(2r+3)}$ for all integers r.

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+3)} = k - \frac{n+1}{(2n+1)(2n+3)},$$

stating the value of k. **[6]**

Question 8(i)

This question was done well providing candidates wrote out a few terms in full. Errors occurred where only the first 2 terms were written out, and/or the last term rather than the final two terms. After cancelling candidates were left with "1" at the beginning, rather than "1 + $\dfrac{1}{3}$" ,erroneously assuming that all the numerical terms cancelled except for the initial '1'. Also this meant that where only the final term was given only one algebraic term remained, "$-\dfrac{1}{2n+3}$" instead of "$-\dfrac{1}{2n+1} - \dfrac{1}{2n+3}$".

Candidates must take care with signs when inserting brackets as errors of this nature occur each year e.g. the two fractions $-\dfrac{1}{2n+1} - \dfrac{1}{2n+3}$ became $-\left[\dfrac{1}{2n+1} - \dfrac{1}{2n+3}\right]$. Deriving the final expression which was given, then introduced another error in the working.

The value of k was requested explicitly and should have been clearly stated, not left within the result to be inferred.

Question 8(ii)

(ii) The sum of the infinite series

$$\frac{1}{(2(n+1)-1)(2(n+1)+3)}+\frac{1}{(2(n+2)-1)(2(n+2)+3)}+\frac{1}{(2(n+3)-1)(2(n+3)+3)}+\dots$$

is $\frac{7}{195}$. Show that n satisfies $28n^2-139n-174=0$ and hence find the value of n. **[5]**

Deriving the given equation was poorly done.

Many candidates did not read the question carefully and simply equated their answer in part (i) to $\frac{7}{195}$.

Those that correctly realised that the given series was the difference between the sum to infinity and their answer from part (i) were able to derive the equation.

By writing out terms of the series a few candidates realised that only two fractions from the initial pair of terms remained, which combined to give the correct equation. Sometimes the $\frac{1}{4}$ was forgotten.

Most candidates, whether they were able to derive the required equation or not, were able to solve the quadratic equation given, although a fair number did not reject $n=-\frac{29}{28}$ as an answer and so lost a mark. Ideally either the use of the quadratic formula or the factorisation of the expression should have been shown, even if the figures had been obtained using a calculator.

Question 9(i)

9 You are given that $\mathbf{M}=\begin{pmatrix}4 & a\\ -6 & -2\end{pmatrix}$ and $\mathbf{N}=\begin{pmatrix}-2 & 6\\ -4a & -14\end{pmatrix}$, where a is a real constant. Find the possible value(s) of a in each of the following cases.

(i) The point $(1, -2)$ is invariant under the transformation represented by matrix **M**. **[2]**

This was done well although some candidates made life hard for themselves by solving $\begin{pmatrix}4 & a\\ -6 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}1\\ -2\end{pmatrix}$ and manipulating the resulting simultaneous equations (often unsuccessfully) eventually inserting $x = 1$ or $y = -2$.

Question 9(ii)

(ii) $(\mathbf{NM}^{-1})^{-1}\mathbf{NM}=\mathbf{N}$. **[4]**

This was very poorly done. There was much 'leapfrogging' of matrices and most candidates showed a complete lack of understanding that matrix multiplication is not commutative. Many candidates derived $M^2 = N$ and then $a = 3$ to no avail, because their first line had been something like $(N^{-1} M)NM = N$ or $NM = N(NM^{-1})$.

Many candidates attempted to evaluate $(NM^{-1})^{-1}NM$ which resulted in a couple of pages of working and only a tiny number succeeded by this method.

Question 9(iii)

(iii) A triangle T_1 has an area of 9 square units. The triangle T_1 is transformed to triangle T_2 by the transformation represented by matrix **M**. The area of triangle T_2 is 144 square units. **[6]**

Most candidates gained the first 4 marks with ease. Only a small minority of candidates realised that the area scale factor might be negative.

There were mistakes made in calculating the determinant of M, with – 8 – 6*a* often seen. The resulting equation was also sometimes manipulated incorrectly, thus 9(6*a* – 8) = 144 became 54*a* – 8 = 144 or 6*a* – 8 = 16 became 6*a* = –24, or similar.

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