Qualification Accredited

Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4776/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4776/01 series overview

The June 2018 series was the final examination opportunity for the 3895-3898, 7895-7898GCE AS/A Level Mathematics (MEI) suite of qualifications. There will be a resit opportunity available in the June 2019 series. The written examination forms 80% of the score for 4776 MEI Numerical Methods, with a coursework element making up the remaining 20%.

Traditionally this unit has formed part of the AS Further Maths certificate, sat in year 12, which then made up 50% of the full A Level Further Maths certificated in year 13. As such, this legacy qualification unit had a different cohort profile to previous years, with the 2017/18 year 12 cohort studying for the reformed linear Maths and Further Maths qualifications.

The majority of the candidates were well prepared for this written paper. The numerical work was generally done efficiently and the candidates performed well on most of the questions on this paper. The candidates could carry out the algorithms correctly, but were less sure of the theoretical conditions underlying these algorithms. Often, having calculated their answer correctly, candidates found it more challenging to provide the required precise or accurate statements needed to comment on the results of their calculations. Some candidates struggled to explain the theory underlying their calculations. There were instances where insufficient detailed was provided in calculations; this might it was difficult to make a reasoned judgement, and statements concerning the interpretation of results were frequently vague, inadequate or incorrect. In a significant number of cases, candidates were making decisions on the first or last terms of their sequences of values. Candidates should make judgements based upon examining all the terms of the sequences that they generate and not just on the extreme values.

Section A overview

This section contains 5 shorter questions testing the candidates on most topics included in the syllabus. The vast majority of the candidates were competent and knowledgeable on the methods being examined.

Question 1

1 The function $f(x)$ has the values shown in the table, correct to six decimal places.

x	1	1.125	1.25	1.5	2
$f(x)$	2.718 282	2.888 277	3.058 835	3.403 298	4.113 250

Use the forward difference method to obtain four estimates of the gradient of $f(x)$ at $x = 1$.

Without doing any further calculation, state the value of $f'(1)$ to the accuracy that appears justified. **[6]**

Most candidates understood how to use the forward differencing method. A number did write the improving approximations in reverse, which made the convergence less obvious, although they were still credited in this case. Some candidates did not give the final approximation to an accuracy that they could justify by either over – or – under - specifying the final value.

Question 2 (i)

2 **(i)** Write down the value of $2^{10} \times 2^{-10}$ and use your calculator to evaluate $2^{10} \div 2^{-10}$. **[2]**

This is answered well by the majority of candidates.

Question 2 (ii)

My calculator displays the value of 2^{-10} as 0.000 976 562.

(ii) Calculate $2^{10} \times 0.000\,976\,562$ and $2^{10} \div 0.000\,976\,562$. **[2]**

This is also well answered in the majority of cases.

Question 2 (iii)

(iii) Explain why your answers to part **(ii)** are different to your answers to part **(i)**. **[1]**

This part was simply looking for a concise statement regarding the fact that 0.000976562 is not an exact value for 2^{-10}. A significant number of candidates did not appreciate this fact and produced reasons that were too diffuse to be creditable.

Question 2 (iv)

When I use my calculator to evaluate $2^{10} + 2^{-10}$ the value 1 024.000 977 is displayed. I obtain the same value when I enter 2^{10} + 0.000 976 562.

(iv) Explain why there is no discrepancy between the values in this case. **[1]**

Most of the candidates discussed the role of calculators and how they stored numbers to more significant figures than they displayed. However, a statement that the loss of accuracy from manual entry only affected the tenth decimal place, was seldom made. This restriction did not affect the displayed values which were the same.

Question 2 (v)

(v) Calculate the relative error in approximating 2^{-10} as 0.000 976 562. **Without** further calculation explain why this is the same as the relative error in the multiplication in part **(ii)**. **[2]**

This was a routine calculation for most candidates. Despite the instruction not to do any further calculations, many candidates did another calculation to show that the two relative errors were the same. Others struggled to provide coherent explanations for the two equal answers. The successful responses referred to the relative error in $A \times B$ = relative error in A + relative error in B and, since there is no error in 2^{10}, the relative errors are the same.

Question 3 (i)

3 The value of $\int_0^1 1.5^x \, dx$ is to be found.

(i) Use the mid-point rule with $h = 1$ and the trapezium rule with $h = 1$ to estimate the value of

$$\int_0^1 1.5^x \, dx .$$ **[3]**

A substantial majority of the candidates calculated estimates of an integral using the mid-point and trapezium rules correctly. The mark scheme allowed, for the mid-point rule, any value in the interval [1.2200000, 1.2247449]. Many candidates rounded their answers to 1.22475. This premature rounding could cause incorrect values in Simpson's rule, if they did not work with a more accurate value stored in their calculator.

Question 3 (ii)

(ii) Calculate a second estimate of the integral using the trapezium rule. **[1]**

Almost all the candidates could use the values of the mid-point and trapezium rules to produce a second estimate of the integral.

Question 3 (iii)

(iii) **Without** doing any further calculation, state the value of $\int_0^1 1.5^x \, dx$ as accurately as you can. **[1]**

On the evidence of the previous estimates, most candidates quoted the answer of 1.2. There were a significant minority of candidates that included more decimal places than could be justified.

Question 3 (iv)

(iv) The mid-point rule estimate of this integral with $h = 0.5$ is 1.231 042 46, correct to eight decimal places. Use this and the estimates found in parts **(i)** and **(ii)** to obtain two Simpson's Rule estimates of $\int_0^1 1.5^x \, dx$ and hence write down the value of the integral to the accuracy that appears justified. **[3]**

The use of the mid-point and trapezium rules to produce estimates of the integral using Simpson's rule was well known and correctly applied. Most candidates achieved the estimated values for Simpson's rule that were contained in the intervals of acceptable answers.

Question 4 (i)

4 The equation $x^2 + 10\,000x + 1 = 0$ has two roots, α and β, where $\beta > \alpha$.

(i) By completing the square, show that

$$\beta = \sqrt{24\,999\,999} - 5\,000. \qquad (*)$$

[2]

The majority of candidates showed that they could complete the square to produce the given answer.

Question 4 (ii)

(ii) Show also that

$$\beta = \frac{-10\,000 + \sqrt{99\,999\,996}}{2}. \qquad (**)$$

[1]

Most candidates used the quadratic formula to obtain the displayed result or multiplied through by 2 in the numerator and denominator in the first result to obtain the second.

Question 4 (iii)

A scientific calculator evaluates (*) as $-0.000\,100\,04$ and (**) as $-0.000\,100\,1$.

(iii) Explain why both of these values are likely to be inaccurate. **[1]**

Only a minority of the candidates noted that the subtraction of near-equal values was the cause of the loss of accuracy, with a significant number of explanatory responses that were not sufficiently convincing to be credited the mark.

Question 4 (iv)

(iv) Show that

$$\beta = -\frac{2}{10\,000 + \sqrt{99\,999\,996}}$$

and hence find an improved approximation for β. **[2]**

The rationalisation of the answer in part (ii) proved to be a challenging manipulation.

There was an elegant answer seen, based upon the fact that the product of the roots was 1, so $\beta = 1/\alpha$.

Question 5 (i)

5 The equation $x - \log_{10} x - 2 = 0$ has a root in the interval $0 < x < 1$.

(i) Use one application of the secant method with $x_0 = 0.5$ and $x_1 = 0.2$ to find an approximation to this root. Explain why the method would fail to find a more accurate approximation to the root in this case. **[3]**

Using the secant method was well understood and usually applied correctly.

Very few candidates noticed that the undefined log of a negative value was the reason why the iteration could not be continued. The majority of candidates focused on the fact that the first iteration produced a negative value that was outside the interval containing the root to explain why the method would fail.

Question 5 (ii)

(ii) Use the secant method with $x_0 = 0.01$ and $x_1 = 0.02$ to obtain the root correct to five decimal places. **[3]**

Many candidates used the secant method for just one or two iterations. This was insufficient to find the root to five decimal places. Rounding after three iterations did give the correct answer, but four iterations would confirm the validity of the answer.

Question 5 (iii)

(iii) State two reasons why the secant method may be preferred to the method of false position. **[2]**

Statements similar to "it is generally faster to converge" and "it is easier to automate as there is no sign check at each stage" were required.

About 55% could give one satisfactory reason and 20% of the candidates could give both reasons. About 25% of the candidates gave no creditable explanation of why the secant method may be preferred to the method of false position.

Section B overview

This section contained 2 questions of a more demanding nature. The majority of candidates gained most of the credit for knowing how to apply the methods. Marks were lost because of the loss of precision or accuracy in the calculations and the weaknesses of the explanations of the theory underlying the methods used.

Question 6 (i)

6 The function f(x) has the values shown in the table.

x	0	1	2	3
f(x)	-2	1	14	85

(i) Construct a table of differences as far as the third difference. **[3]**

Virtually all the candidates completed a correct difference table.

Question 6 (ii)

(ii) Construct the Newton interpolating polynomial of degree 3, giving your answer in the form $ax^3 + bx^2 + cx + d$. **[5]**

Most then went on to apply Newton's interpolating formula correctly.

Question 6 (iii) (A)

(iii) Use your answer to **(ii)** to find

(A) an estimate of the gradient of f(x) at $x = 2.5$, **[3]**

Estimating the gradient was well understood, except significant minority of candidates forgot to differentiate their polynomial. Those candidates with an incorrect polynomial could get credit for the method, but not for the final answer.

Question 6 (iii) (B)

(B) an estimate of $\int_0^3 \mathrm{f}(x)\,\mathrm{d}x$. **[3]**

All candidates were able to integrate their polynomials and substituted the 2.5 value. Again some credit was given to candidates with an incorrect cubic polynomial.

Question 6 (iv)

(iv) It is subsequently found that f(4) = 358. Determine whether this new information casts doubt on the reliability of your answers to parts **(iii) (A)** and **(B)**. **[4]**

This part was less well answered; many candidates evaluated their f(4) correctly and pointed out that this was different from the value of the f(4) provided, but felt that their functions were still reasonable as they fitted the values used. They did not appreciating that a quartic or higher degree polynomial could have an entirely different shape, thereby invalidating the estimates for the gradient and the integral. A few candidates realised that the extra given value allowed them to extend their difference table. These candidates now had more evidence and could argue with more conviction on the unreliability of the gradient and integral.

Question 7 (i)

7 **(i)** Show that the equation $x^5 - 6x + 4 = 0$ has a root, α, such that $-2 < \alpha < -1$. **[2]**

Nearly all the candidates could show that a change of sign indicated the presence of a root in the given interval.

Question 7 (ii)

(ii) Show numerically that using the Newton-Raphson method with $x_0 = -1$ fails to find α. **[3]**

This question was well understood and many candidates correctly applied the requested method. A number of candidates did not carry out sufficient iterations to show that the approximation was not converging to a value in the required interval. Some only completed one or two iterations before making a decision based upon inconclusive evidence.

Question 7 (iii)

(iii) Use the iteration $x_{r+1} = \sqrt[5]{6x_r - 4}$ with $x_0 = -1$ to find α correct to 6 decimal places. **[3]**

Almost all candidates used the given iteration to find an approximate value of *x* in the required interval. The request for 6 decimal place accuracy usually produced the correct answer of –1.700040. Some candidates did not round the last 2 decimal places correctly, whilst others dropped the final 0.

Question 7 (iv)

There is another root of the equation, β, such that $0 < \beta < 1$.

The following iteration, with $x_0 = 0$, was used to find β.

$$x_{r+1} = \frac{x_r^5 + 4}{6} \qquad (*)$$

The results are shown in the table below.

The difference between successive iterates is shown in the third column, and the ratio of these differences is shown in the fourth column. The values in the fourth column are given to two decimal places.

r	x_r	$x_{r+1} - x_r$	$\frac{x_{r+2} - x_{r+1}}{x_{r+1} - x_r}$
0	0	0.666 667	0.03
1	0.666 667	0.021 948	0.18
2	0.688 615	0.003 858	0.19
3	0.692 473	0.000 731	0.19
4	0.693 204	0.000 141	0.19
5	0.693 345	0.000 027	0.19
6	0.693 372	0.000 005	0.20
7	0.693 377	0.000 001	–
8	0.693 378	0	
9	0.693 378		

(iv) What do the values in the fourth column tell you about the convergence of the iteration (*) with $x_0 = 0$? Justify your answer. **[2]**

There were a significant number of candidates who confused the order of convergence of the iteration with the order of a method of numerical integration. In other words, they felt that a common ratio of 0.2 was close enough to 0.25 to describe the convergence as second order.

Question 7 (v)

(v) Use the Newton-Raphson method with $x_0 = 0$ to find β correct to 8 decimal places, showing the iterates in the table in the Printed Answer Book. Hence complete the table. **[6]**

The majority of the candidates received the method marks for this iteration but very few provided sufficiently accurate results (particularly for the ratio of differences) to gain full marks for this section.

Question 7 (vi)

(vi) What do your values in the fourth column tell you about the convergence of this iteration? Justify your answer. **[2]**

Asking for a decision on the convergence of this iteration only served to enforce the fact that many of the candidates misunderstand the concept of convergence. Very few candidates were able to discuss and justify the convergence of this method.

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