Qualification Accredited

Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4758/01 Summer 2018 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4758/01 series overview

This was the final assessment series for the unitised 3895-3898, 7895-7898 GCE Mathematics (MEI) specification. There will be a resit opportunity in the summer 2019.

Differential Equations (DE) 4758 is an A2 GCE component taken as part of the Mathematics (MEI) specification, occasionally used as the second optional component in AS Further Mathematics (3896), but generally as one of the four optional components in A Level Further Mathematics (7896). A few candidates are also studying AS Further Mathematics (Additional) (3897) or A Level Further Mathematics (Additional) (7897), and in this case the grading optimisation process will determine which qualification this component will contribute towards.

Candidates are required to answer three of the four questions on this paper. Most candidates attempted Question 1 and Question 4 (Q1 98% and Q4 82% of candidates), and then one of the other two questions. Question 3 was the least popular (Q2 69% and Q3 52%). There were a small percentage of candidates that answered more than the required three questions: in these cases all questions were marked and the best three scores used. Generally, candidates are better advised to focus all their time on three specific questions rather than attempting all four questions in a time designed for three full answers; any time at the end would be better used to refine their earlier answers rather than attempting the extra question.

The general standard of the responses was high, with many candidates gaining full marks on more than one question. Candidates showed a sound knowledge of the methods and techniques involved. There were some arithmetical errors in basic algebra which proved costly for some candidates, but these appeared to be careless mistakes rather than an indication of any fundamental weakness; overall the standard of accuracy was very good.

Question 1(i)

1 In this question, you may assume that $t^k e^{-t} \to 0$ as $t \to \infty$ for any constant k.

The differential equation $4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = f(t)$ is to be solved for $t \geqslant 0$.

Firstly consider the case $f(t) = 9t^2 - 3t - 1$.

(i) Find the general solution for x in terms of t. **[9]**

Almost all candidates were able to apply the method for the solution of a second order differential equation. The forms of the complementary function and the particular integral were known and there were very few arithmetical errors.

Question 1(ii)

You are given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$.

(ii) Find the particular solution. **[4]**

This was answered well with the majority of candidates scoring full marks.

Question 1(iii)

(iii) Show that x is positive for all values of $t \geqslant 0$. **[3]**

Candidates were required to show that both the exponential part and the quadratic part of the particular solution found in part (ii) were positive. This was done with varying degrees of success. The exponential term $(6t+2)e^{-1.5t}$ is always positive for positive values of *t*. Some candidates seemed to think that the behaviour of the exponential function for large values of *t* was relevant here. There are several ways of showing that the quadratic term $t^2 - 3t + 3$ is positive: completing the square and noting that the constant term 0.75 is positive, or finding the turning point and noting that it was a positive quadratic with a minimum in the first quadrant, or using the discriminant to show that the quadratic does not cross the *x*–axis. Many candidates who adopted the last of these approaches did not realise that they needed to say why the curve was wholly above and not wholly below the axis.

Question 1(iv)

Now consider the case $f(t) = -48\sin 2t - 14\cos 2t$.

(iv) Find the general solution for x in terms of t. **[6]**

The majority of candidates earned full marks in this part. The only errors were arithmetical slips in solving the simultaneous equations when finding the particular integral.

Question 1(v)

(v) Describe the behaviour of x for large values of t. **[2]**

A comment on the behaviour of the exponential part of the solution found in part (iv), for large values of *t*, and a statement that the remaining part of the solution was oscillatory was required for the first mark. A second mark was credited for noting the amplitude of the resulting motion.

Question 2(i)

2 **Take g as 10 in this question.**

A particle P of mass 0.1 kg is in a liquid and is projected vertically downwards. At time t s, the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and the depth of P below its point of projection, O, is x m. The only forces on P are its weight and a resistance force R N. A scientist investigates two different models for R.

In the first model, the resistance is given by $R = 0.2v$ and the initial speed of P is $2\,\mathrm{m\,s^{-1}}$.

(i) Use this information to form a differential equation involving v and t. Solve the differential equation to show that $v = 5 - 3e^{-2t}$. **[7]**

Almost all candidates scored full marks in this part. The most popular method was by separation of variables.

Question 2(ii)

(ii) Sketch the graph of v against t. **[2]**

The majority of candidates successfully completing part (i) could provide an appropriate sketch, although a few candidates only scored one out of the two marks for not labelling either the x-intercept or the asymptote.

Question 2(iii)

(iii) Find an expression for x in terms of t and hence find the depth of P below O when its speed is three-quarters of its terminal speed. **[7]**

The majority answered this part well, although careless arithmetic errors prevented some candidates for progressing through to a successful conclusion.

Question 2(iv)

In the second model, the resistance is given by $R = 0.0625v^2$ and the initial speed of P is again $2\,\mathrm{m\,s^{-1}}$.

(iv) Find v in terms of x. **[6]**

The responses to this part were variable in quality. The best solutions showed a careful and accurate application of integration by separation of variables. A significant minority of candidates made little progress in the integration. Others attempted to find *v* in terms of *t*.

Question 2(v)

(v) State the terminal speed of P and find the depth of P below O when its speed is three-quarters of its terminal speed. **[2]**

Generally, those candidates that had progressed through the question could pick up at least one mark here, with around one half of the candidates gaining full marks.

Question 3(a)(i)

3 **(a)** A curve in the x-y plane satisfies the differential equation $\frac{dy}{dx} - \frac{2y}{x} = x^k \sin 2x$,

where k is a constant and $x > 0$.

Firstly consider the case $k = 3$.

(i) Find the general solution for y in terms of x. **[7]**

Responses to this part were almost always correct, although some careless arithmetic errors were seen. A few candidates did not multiply through the right-hand side by their correct integrating factor. This led to multiple applications of integration of parts, which did not deter them.

Question 3(a)(ii)

(ii) Given that $y = 0$ when $x = \frac{1}{4}\pi$, find the exact value of y when $x = \frac{1}{2}\pi$. **[4]**

The method was clearly known, but again there were many arithmetical errors that cost full credit.

Question 3(a)(iii)

Now consider the case $k = 2.5$.

(iii) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 0.5$, to estimate y when $x = 0.8$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[5]**

This was a routine request and the responses reflected this.

Question 3(b)(ii)

(ii) In your Answer Book, sketch on the given axes the isoclines for the cases $\frac{dy}{dx} = m$ for $m = 0, \pm 1, \pm 2$. Use these isoclines to draw a tangent field. **[3]**

For full marks, candidates needed to sketch five isoclines, each one a parabola, and mark on each one a few direction indicators. Some candidates attempted to mark direction indicators without sketching the isoclines. Other candidates sketched only two or three of the isoclines. A common error was to assume that a parabola with a positive *y*-intercept had positive direction indicators.

Question 3(b)(iii)

(iii) Sketch the solution curve through (0, 1) and the solution curve through (1, 0). **[3]**

There were some very pleasing sketches of the two solution curves. A common error was to have the reflections of the correct curves, in the *y*-axis. This error usually resulted from incorrect direction indicators in part (ii).

Question 4(i)

4 The simultaneous differential equations

$$\frac{dx}{dt} = 7x + 2y + 13e^{4t},$$

$$\frac{dy}{dt} = -9x + y + e^{7t}$$

are to be solved.

(i) Eliminate x to obtain a second order differential equation for y in terms of t. Hence find the general solution for y. **[12]**

This is a standard question on the solution of simultaneous differential equations, but with a slight twist. The instruction to eliminate *x* to obtain a second order differential equation for *y* in terms of *t* was less familiar to candidates and many embarked on the more familiar route of eliminating *y* to find a differential equation for *x* in terms of *t*. Having realised their error, the majority of candidates who had pursued this route crossed out their work and began again. Others abandoned the question altogether. A very small minority realised that they could use their solution for *x* in the second given differential equation and thereby find *y*.

Question 4(ii)

(ii) Given that $y = -3$ and $\frac{dy}{dt} = 60$ when $t = 0$, find the particular solution for y. **[4]**

Careless arithmetic in the substitutions prevented candidates from gaining full credit

Question 4(iii)

(iii) Find the corresponding particular solution for x. **[2]**

Over half of the candidates gained the first method mark, but errors in the algebraic manipulation resulted in candidates missing out on the accuracy mark.

Question 4(iv)

(iv) Find the smallest positive value of t for which $y = 0$. **[4]**

Most candidates equated their solution to part (ii) to zero, but only a minority were able to solve the resulting trigonometric equation: $10\cos 3t + 24\sin 3t = 13$. The most common method of solution was to equate the right-hand side to $R\sin(3t + \alpha)$.

Question 4(v)

(v) Show that $\frac{y}{x} \to 0$ as $t \to \infty$. **[2]**

Only a minority of candidates were able to make any convincing progress in this part. Many of the arguments developed were too vague to be awarded full credit.

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