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Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4768/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4768/01 series overview

The overall quality of the candidates for this paper was very high and a good level of preparation was evident. Almost all candidates seem able to carry out standard calculations accurately, displaying confident use of technology and also showing sufficient working. Candidates should be reminded however of the need to use an appropriate degree of accuracy, which for this component is usually three or four significant figures. They should understand that just because their calculator shows more figures, this does not mean that they are relevant or correct in every context.

The procedure for hypothesis testing was also well understood, including the need to state hypotheses and the conclusion correctly. Only a small minority lost marks for using an incorrect form of words or omitting the context. An exception to this was the χ^2 test, where the context was often left out.

The notation for combinations of random variables however is an area that requires some attention. Although the correct calculations often suggested that candidates knew the difference between (for example) $2A$ and $A_1 + A_2$, this was often not expressed using correct notation. Making the distinction explicit would not only help candidates select the correct variance calculation, but is also an essential part of correct mathematical communication, which should be given more emphasis at this level.

The weakest topic in this component remains the understanding of the Central Limit Theorem, specifically its exact statement, when it can be applied and when it is not needed. Linked to this is the understanding of the sample mean as a random variable and the distinction between the distributions of the sample mean and the original population. Simulations using technology can help candidates understand this distinction.

Although candidates generally showed good knowledge of standard definitions, conditions and assumptions, they need to be careful to answer precisely the question being asked, rather than simply repeating standard statements. For example, they should note whether a questions asks whether a certain result can be used or whether it is required.

Key

AfL Guidance to offer for future teaching and learning practice.

Misconception

Question 1(a)(i)

1 **(a)** A psychology experiment is designed to investigate whether the colour of the paper on which text is printed affects reading speed. Eight randomly selected participants are given two pieces of text to read, one printed on white paper and one printed on pale green paper. The two texts are different, but contain approximately the same number of words and are of equal complexity.

The table shows the length of time, in seconds, that each participant took to read each piece of text. The times have been recorded to the nearest second.

Participant	A	B	C	D	E	F	G	H
White paper	63	55	78	61	48	63	81	57
Green paper	51	61	70	59	45	65	66	50

(i) Explain why a paired test is appropriate in this context. **[1]**

Most candidates understood that one advantage of a paired test is that it eliminates differences between individual participants (in this case, differences in reading speed). A paired test can only be carried out if there are two pieces of information for each participant. Unfortunately, many candidates left out this important point.

Question 1(a)(ii)

(ii) Carry out a *t* test at the 5% significance level to test whether there is a difference in the population mean times taken to read the two texts. You may assume that the distributional assumptions for the *t* test hold. **[9]**

The calculations for the *t* test were generally done well and with sufficient detail. Many candidates scored full marks on this question, although a sizeable minority did lose one or two marks for not referring to the 'population' or 'mean times'. It is important to realise that a paired test is for the mean of the population of differences, not for the difference of the two population means. This should be reflected in the phrasing in the hypotheses, for example '$\mu = 0$, where μ is the mean of the population of the differences in reading times' and not 'where μ is the difference between the population reading times'.

Question 1(b)(i)

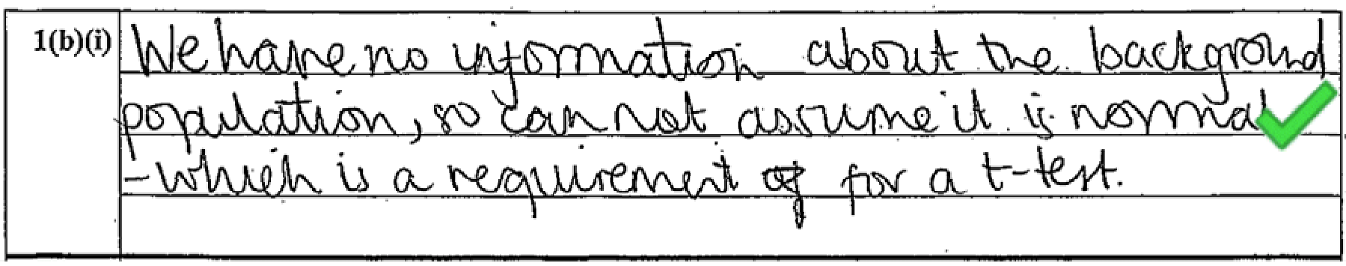
(b) A psychology textbook claims that the typical capacity for short term memory is seven items. A student believes that among college students it is higher than this. To test his hypothesis, he selects a random sample of ten college students. Each student is shown a list of thirty words and asked to repeat them two minutes later. The number of words each student could remember is given in the table.

Student	A	B	C	D	E	F	G	H	I	J
Number of words	10	15	2	18	13	6	21	3	16	5

(i) Explain why a t test would not be appropriate in this situation. **[1]**

Almost all candidates scored this mark, which was given for stating that the underlying population distribution was unknown.

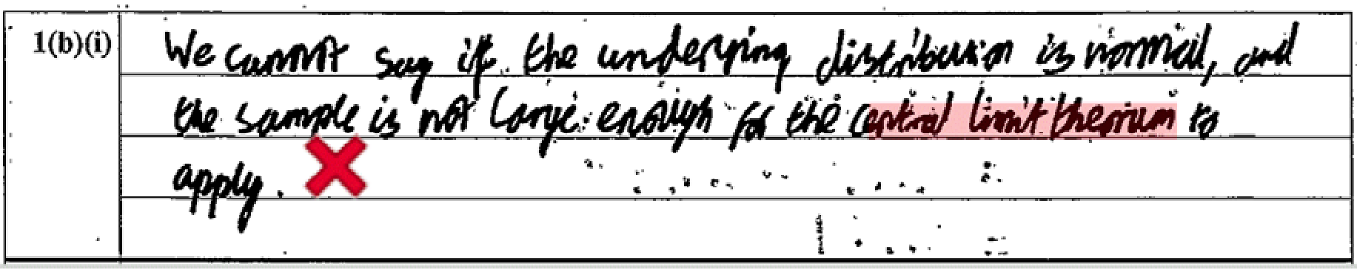
Exemplar 1



1(b)(i) We have no information about the background population, so cannot assume it is normal - which is a requirement of for a t-test.

The best answers, such as Exemplar 1, stated clearly the requirement for using the *t* test and explained why it is not met in this case.

Exemplar 2



1(b)(i) We cannot say if the underlying distribution is normal, and the sample is not large enough for the central limit theorem to apply.

Some answers, such as Exemplar 2, reveal the misunderstanding of the Central Limit Theorem (CLT) that was seen more widely in Q2 (iii). The CLT tells us about the distribution of the sample mean, while a *t* test requires a Normally distributed population, which cannot be changed by taking a larger sample size.

Question 1(b)(ii)

(ii) Use a Wilcoxon test with a 5% significance level to test whether, among college students, the average number of words remembered is greater than 7. **[9]**

Question 2(i)

2 The number of typing errors on a page is often given as an example of a Poisson distribution.

Elaine, who is a typist, wants to investigate whether this is the case for her typing. She picks a random sample of 100 full pages she has typed over the past year and counts the number of typing errors on each page. Her results are recorded in the table.

Number of typing errors on a page	0	1	2	3	4	5	6	$\geqslant 7$
Number of pages	13	15	16	22	22	8	4	0

(i) Use the data in the table to estimate the population mean and variance of the number of typing errors per full page. Comment whether, in the light of these values, a Poisson distribution might be a suitable model for the number of typing errors on a full page of Elaine's typing. **[2]**

Most candidates knew what was required in this question, and the fact that the Poisson distribution has mean equal to the variance seems well understood. A minority of candidates lost a mark for finding the sample variance, rather than the unbiased estimate of the population variance, or by giving their answer to too many significant figures.

Question 2(ii)

(ii) The table in the Answer Book shows some of the expected frequencies and contributions to the test statistic for a χ^2 test for the goodness of fit of a Poisson model for the number of errors on a full page of Elaine's typing. Calculate the missing expected frequencies and hence complete the test using a 5% significance level. **[10]**

There was a good number of correct answers to this question. The fact that two of the rows of the table needed to be combined presented a difficulty to a sizeable minority of candidates. Some forgot to combine rows altogether. Others stated that the rows needed to be combined, but then did not recalculate the relevant contribution. It was common to see the rows combined for the purposes of the calculation, but then the original number of rows used for determining the degrees of freedom (or vice versa).

An additional difficulty arose for candidates who calculated the expected frequency for the final row incorrectly. The most common error was to use $P(X = 7)$ rather than $P(X \geq 7)$. This results in the last three rows needing to be combined and further confusion in determining degrees of freedom.

When calculating the degrees of freedom some candidates subtracted 2 from the usual $n - 1$, possibly because they were asked to calculate both mean and variance in part (i). However, the Poisson distribution only has one parameter, so subtracting 1 is correct here.

This question highlighted the importance of showing detailed working. For example, if a candidate mistakenly combined the last three rows, but then clearly stated 'dof = 6 -1 -1' they were able to get some follow-through marks that they would not gain if they just wrote 'dof = 4'. Similarly, an incorrect answer for the χ^2 statistic could gain one out of the two marks provided a clear calculation (adding the contributions) was shown.

Finally, although almost all candidates included context in their conclusion of the hypothesis tests in Q1, in this question many just wrote 'There is sufficient evidence that the model does not fit the data', which cost them the final mark.

Question 2(iii)

(iii) Construct a 95% confidence interval for the mean number of typing errors per full page of Elaine's typing. Explain whether your calculation relies on the Central Limit Theorem. **[5]**

The calculation for the confidence interval was generally done correctly. Almost all candidates attempted to explain how or why they used the Central Limit Theorem in their calculation, however from the responses it seems that the precise statement of the theorem is not very well understood.

It was good to see that only a small number of candidates thought that the CLT says something about the variance of the sample mean; the fact that the variance needs to be divided by *n* is true in general, and not a part of the statement of the CLT. A more common mistake was to claim that, for a large enough sample, the underlying population is approximately Normally distributed.

Such candidates would benefit from a better understanding of the sample mean as a random variable and how its distribution changes with the sample size. A possible way to do this is to use technology to generate lots of samples from a given distribution and build up the distribution of the means.

Those candidates who did know the statement for the CLT were often not able to express the answer to the actual question correctly. The calculation relies on the CLT because the underlying distribution is not Normal (as it is a discrete distribution), so the distribution of the sample means needs to be approximated in order to calculate the confidence interval. If the underlying distribution had been Normal, CLT would not have been required, regardless of the sample size. Many candidates explained why the CLT can be used (because the sample size is large) rather than why it is required.

Central Limit Theorem

The Central Limit Theorem (CLT) states that, when the sample size is sufficiently large, the distribution of the sample means is approximately Normal, regardless of the underlying population distribution.

The CLT only needs to be used if the underlying population is not known to be Normal.

If the underlying population is Normal, the CLT is not required as the distribution of the sample means is exactly Normal for any sample size.

Question 3(i)

3 The table shows the mean and standard deviation of the number of calories in a single apple, a single banana and a single strawberry. It can be assumed that the number of calories in each type of fruit is Normally distributed.

Fruit	Apple	Banana	Strawberry
Mean (calories)	97.0	112.5	5.5
Standard deviation (calories)	6.3	7.5	1.3

(i) Find the probability that a randomly selected banana contains more than 100 calories. **[2]**

Question 3(ii)

(ii) Write down the probability distribution of the number of calories in half a banana.

Find the probability that one apple contains at least 40 more calories than half a banana. **[6]**

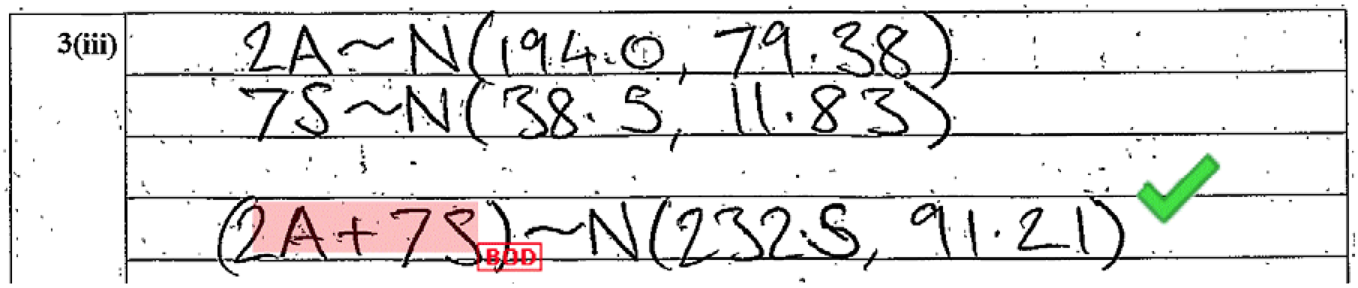
There were many correct answers to this question. It was quite common to see the variance for half a banana being divided by 2 instead of by 4, which had a knock-on effect on the final answer.

Question 3(iii)

(iii) Vesna makes a smoothie using two apples and seven strawberries. Assuming that the fruits are randomly and independently selected, find the probability that Vesna's smoothie contains more than 250 calories. **[5]**

In this question the variances needed to be multiplied by 2 and 7 rather than 4 and 49, before adding. This is because there are two different, independent apples, rather than two copies of the same apple (and similarly for strawberries). The correct notation of the overall random variable is $T = A_1 + A_2 + S_1 + \ldots + S_7$ rather than $T = 2A + 7S$. The latter was condoned where it led to a correct answer, but is strictly speaking wrong.

Exemplar 3



Exemplar 3 uses incorrect notation, but is given benefit of the doubt because of finding the variance correctly.

Exemplar 4

3(iii)

$V = 2A + 7S$ M0 $\quad S \sim N(5.5, 1.3^2)$

$E(V) = 2 \times E(A) + 7E(S) = 2 \times 97 + 7 \times 5.5 = 232.5$ B1

$Var(V) = 2^2 Var(A) + 7^2 Var(S) = 4 \times 6.3^2 + 49 \times 1.3^2 = 241.57$ M0

Exemplar 4 uses incorrect notation and finds variance incorrectly, so is only given one mark for the mean.

Exemplar 5

3(iii)

$A_1 + A_2 + S_1 + \ldots + S_7$

$P($ [illegible] $> 250)$

mean $= 2 \times 97 + 7 \times 5.5$

$= 232.5$

variance $= 2 \times 6.3^2 + 7 \times 1.3^2$ ✓

$= 91.21$

Exemplar 5 uses correct notation.

Candidates should be encouraged to define their random variables correctly. This should help them decide whether the variance needs to be multiplied by 7 or by 49, for example.

Question 3(iv)(A)

'Red Ripple' is a smoothie that is sold in bottles. The amount, x millilitres, of drink in 12 randomly selected bottles of 'Red Ripple' smoothie is measured, and the results are summarised as follows:

$$\Sigma x = 2184, \quad \Sigma x^2 = 397\,851.$$

The amount of drink in a bottle is assumed to be Normally distributed.

(iv) (*A*) Construct a 95% confidence interval for the mean amount of drink in a bottle of 'Red Ripple'. **[5]**

The process for construction confidence intervals seems well understood in general. A sizeable minority of candidates used the Normal rather than the *t* distribution (possibly because it was mentioned in the question). The *t* distribution is required here because the population variance is unknown.

The fact that the bounds of the confidence interval round to 178.35 and 185.65 seems to have caused many candidates to give their answers to five significant figures, hence losing the final mark.

Question 3(iv)(B)

(*B*) Explain what is meant by a 95% confidence interval in this context. **[1]**

The meaning of a 95% confidence interval seems to be very well understood. To gain the mark, candidates needed to make it clear that they were referring to confidence intervals 'constructed in this way', rather than 'all confidence intervals'.

Question 4(i)(A)

4 The length of time, in minutes, that I have to wait in the queue for coffee in the college canteen is modelled by the random variable T with cumulative distribution function

$$\mathrm{F}(t) = \begin{cases} 0 & t < 0, \\ \frac{1}{3}t^2 & 0 \leqslant t \leqslant 1, \\ -\frac{1}{6}t^2 + t - \frac{1}{2} & 1 < t \leqslant 3, \\ 1 & t > 3. \end{cases}$$

(i) Use this model to find

(*A*) the probability that I have to wait for more than 2 minutes, **[2]**

Question 4(i)(B)

(*B*) the median waiting time. **[3]**

A majority of candidates knew what to do here and were able to select the correct equation to use. The best answers showed explicitly that the median has to be greater than 1 (since F(1) < 0.5). Some candidates seemed to find two "medians", one for $t < 1$ and one for $t > 1$.

Question 4(ii)

(ii) Find the probability density function of T. **[3]**

Question 4(iii)

(iii) Show that the expected value of T is $\frac{4}{3}$. **[4]**

Since this is a 'show that' question, all the details need to be clearly shown (the result of the integration, the substitution of limits and some calculation steps leading to the final number).

Question 4(iv)

You are given that the variance of T is $\frac{7}{18}$.

(iv) I record the time I have to wait in the queue on 30 randomly selected days. Calculate an estimate of the probability that the mean of these 30 waiting times is greater than 1.5 minutes. **[4]**

This final question was very well answered, with almost all candidates realising that the Central Limit Theorem allows them to approximate the distribution of the sample mean to Normal, using the mean from part (iii) and dividing the given variance by 30.

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