



GCE

Mathematics (MEI)

Unit **4777**: Numerical Computation

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

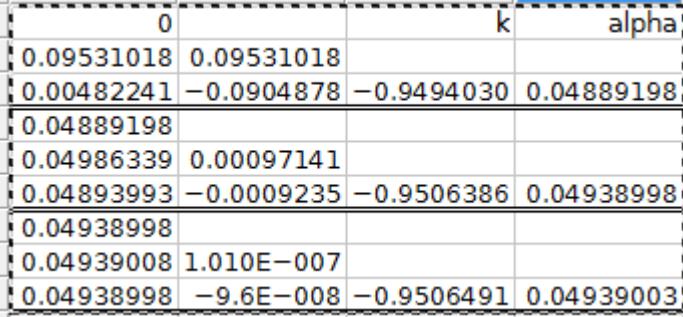
All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scores	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Question		Answer	Marks	Guidance																		
1	(i)	<p>Approx error in x_2 is $k^2 \varepsilon$</p> $x_0 = \alpha + \varepsilon$ $x_1 = \alpha + k\varepsilon \quad x_1 - x_0 = (k-1)\varepsilon$ $x_2 = \alpha + k^2 \varepsilon \quad x_2 - x_1 = k(k-1)\varepsilon \quad (x_2 - x_1)/(x_1 - x_0) = k$ <p>Hence given expressions for α</p>	B1 B1 B1 B1B1 B1 [6]	Expression for $x_{0,1,2}$ For Δx_0 For $\Delta x_1, k$																		
1	(ii)	<p>Let $f(x) = \sin x + e^x$. Then $f'(x) = \cos x + e^x$. Note $f(0) = 1$, $f'(0) > 0$. $f'(x) = 0$ requires $\cos x = -e^x$, but $e^x > 1$ for $x > 0$. So f has no turning points for $x > 0$, and $f(x) = c > 1$ has a unique solution. Sketch of $\sin x + e^x$ to confirm this</p>	B1 B1B1 B1 G1 [5]																			
	(iii)	<p>Iterations with $c = 1.1$, very slow e.g.:</p> <table border="1"> <tr><td>0</td><td>0.09531</td><td>0.004822</td><td>0.090917</td><td>0.009167</td><td>0.086942</td></tr> <tr><td>0.1</td><td>0.000167</td><td>0.095159</td><td>0.004972</td><td>0.09078</td><td>0.009302</td></tr> <tr><td>0.05</td><td>0.04881</td><td>0.049941</td><td>0.048866</td><td>0.049888</td><td>0.048916</td></tr> </table> <p>Set up and use method in part (i). E.g.:</p>	0	0.09531	0.004822	0.090917	0.009167	0.086942	0.1	0.000167	0.095159	0.004972	0.09078	0.009302	0.05	0.04881	0.049941	0.048866	0.049888	0.048916	M1A1 B1 for a 2nd example	
0	0.09531	0.004822	0.090917	0.009167	0.086942																	
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		 <p>Root is 0.04939 to 4 sf</p>	M2 for a correct spreadsheet A1A1A1 for three steps. A1 final answer [9]																			

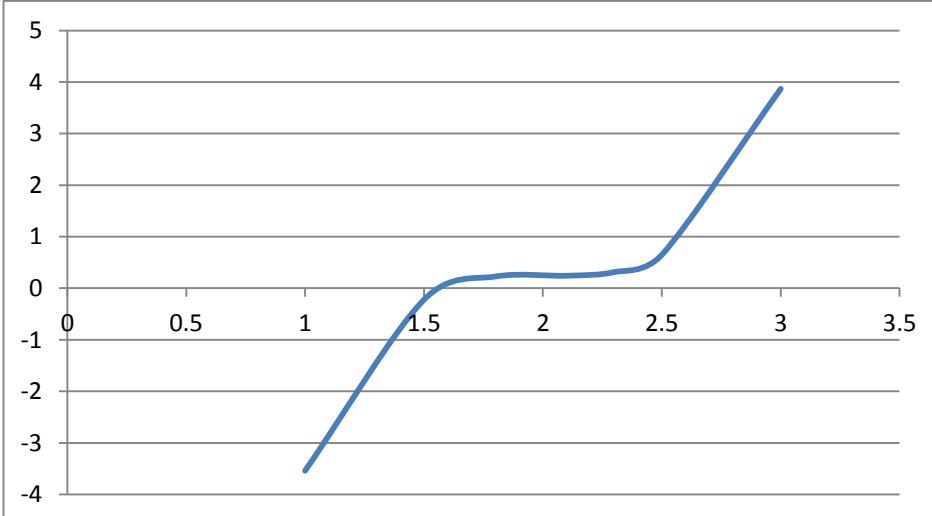
Question		Answer	Marks	Guidance												
1	(iv)	<p>At least two examples with larger c, illustrating faster convergence. E.g. $c = 4$:</p> <table border="1"><tr><td>1</td><td>1.150106</td><td>1.127262</td><td>1.130356</td><td>1.129929</td><td>1.129988</td></tr></table> <p>E.g. $c = 8$:</p> <table border="1"><tr><td>2</td><td>1.958784</td><td>1.956472</td><td>1.956349</td><td>1.956343</td><td>1.956342</td></tr></table>	1	1.150106	1.127262	1.130356	1.129929	1.129988	2	1.958784	1.956472	1.956349	1.956343	1.956342	M1A1 M1A1 [4]	Assessment
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			[24]													

Question		Answer										Marks	Guidance																																																																																																																																																																																										
2	(i)	<p>Set up as $\int_{-h}^h f(x)dx = af(-\alpha) + af(\alpha)$</p> <p>$f(x) = 1 \quad 2h = 2a \quad a = h$</p> <p>$f(x) = x, (\text{and } x^3) \quad 0 = 0$</p> <p>$f(x) = x^2 \quad 2h^3/3 = 2a \alpha^2 \quad \alpha = h / \sqrt{3}$</p> <p>Rule is exact up to cubics</p>										M1A1 B1 B1 B1 B1	Answer given Answer given No reason required																																																																																																																																																																																										
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2	(ii)	<table border="1"> <thead> <tr> <th>a</th><th>b</th><th>m</th><th>h</th><th>L</th><th>R</th><th>f(L)</th><th>F(R)</th><th>G</th><th>Total</th></tr> </thead> <tbody> <tr><td>0</td><td>1</td><td>0.5</td><td>0.5</td><td>0.211325</td><td>0.788675</td><td>0.999004</td><td>0.849138</td><td>0.924071</td><td>0.924071</td></tr> <tr><td>0</td><td>0.5</td><td>0.25</td><td>0.25</td><td>0.105662</td><td>0.394338</td><td>0.999938</td><td>0.988124</td><td>0.497016</td><td></td></tr> <tr><td>0.5</td><td>1</td><td>0.75</td><td>0.25</td><td>0.605662</td><td>0.894338</td><td>0.938828</td><td>0.78093</td><td>0.429939</td><td>0.926955</td></tr> <tr><td>0</td><td>0.25</td><td>0.125</td><td>0.125</td><td>0.052831</td><td>0.197169</td><td>0.999996</td><td>0.999245</td><td>0.249905</td><td></td></tr> <tr><td>0.25</td><td>0.5</td><td>0.375</td><td>0.125</td><td>0.302831</td><td>0.447169</td><td>0.995821</td><td>0.980588</td><td>0.247051</td><td></td></tr> <tr><td>0.5</td><td>0.75</td><td>0.625</td><td>0.125</td><td>0.552831</td><td>0.697169</td><td>0.956334</td><td>0.899391</td><td>0.231966</td><td></td></tr> <tr><td>0.75</td><td>1</td><td>0.875</td><td>0.125</td><td>0.802831</td><td>0.947169</td><td>0.840535</td><td>0.744356</td><td>0.198111</td><td>0.927033</td></tr> <tr><td>0</td><td>0.125</td><td>0.0625</td><td>0.0625</td><td>0.026416</td><td>0.098584</td><td></td><td>1</td><td>0.999953</td><td>0.124997</td><td></td><td></td><td></td></tr> <tr><td>0.125</td><td>0.25</td><td>0.1875</td><td>0.0625</td><td>0.151416</td><td>0.223584</td><td>0.999737</td><td>0.998753</td><td>0.124906</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.25</td><td>0.375</td><td>0.3125</td><td>0.0625</td><td>0.276416</td><td>0.348584</td><td>0.997094</td><td>0.992698</td><td>0.124362</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.375</td><td>0.5</td><td>0.4375</td><td>0.0625</td><td>0.401416</td><td>0.473584</td><td>0.987265</td><td>0.97576</td><td>0.122689</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.5</td><td>0.625</td><td>0.5625</td><td>0.0625</td><td>0.526416</td><td>0.598584</td><td>0.963683</td><td>0.941395</td><td>0.119067</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.625</td><td>0.75</td><td>0.6875</td><td>0.0625</td><td>0.651416</td><td>0.723584</td><td>0.920549</td><td>0.885917</td><td>0.112904</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.75</td><td>0.875</td><td>0.8125</td><td>0.0625</td><td>0.776416</td><td>0.848584</td><td>0.856425</td><td>0.811498</td><td>0.104245</td><td></td><td></td><td></td><td></td></tr> <tr><td>0.875</td><td>1</td><td>0.9375</td><td>0.0625</td><td>0.901416</td><td>0.973584</td><td>0.776095</td><td>0.725772</td><td>0.093867</td><td>0.927037</td><td></td><td></td><td></td></tr> </tbody> </table> <p>Integral is 0.9270 to 4 dp.</p>														a	b	m	h	L	R	f(L)	F(R)	G	Total	0	1	0.5	0.5	0.211325	0.788675	0.999004	0.849138	0.924071	0.924071	0	0.5	0.25	0.25	0.105662	0.394338	0.999938	0.988124	0.497016		0.5	1	0.75	0.25	0.605662	0.894338	0.938828	0.78093	0.429939	0.926955	0	0.25	0.125	0.125	0.052831	0.197169	0.999996	0.999245	0.249905		0.25	0.5	0.375	0.125	0.302831	0.447169	0.995821	0.980588	0.247051		0.5	0.75	0.625	0.125	0.552831	0.697169	0.956334	0.899391	0.231966		0.75	1	0.875	0.125	0.802831	0.947169	0.840535	0.744356	0.198111	0.927033	0	0.125	0.0625	0.0625	0.026416	0.098584		1	0.999953	0.124997				0.125	0.25	0.1875	0.0625	0.151416	0.223584	0.999737	0.998753	0.124906					0.25	0.375	0.3125	0.0625	0.276416	0.348584	0.997094	0.992698	0.124362					0.375	0.5	0.4375	0.0625	0.401416	0.473584	0.987265	0.97576	0.122689					0.5	0.625	0.5625	0.0625	0.526416	0.598584	0.963683	0.941395	0.119067					0.625	0.75	0.6875	0.0625	0.651416	0.723584	0.920549	0.885917	0.112904					0.75	0.875	0.8125	0.0625	0.776416	0.848584	0.856425	0.811498	0.104245					0.875	1	0.9375	0.0625	0.901416	0.973584	0.776095	0.725772	0.093867	0.927037			
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		<p>Ratios of differences for these totals are 0.027212 and 0.047136</p> <p>Theory indicates $1/16 = 0.0625$ for a 4th order method. Not close (but approaches in the limit).</p>										M1A1 E1E1 [4]																																																																																																																																																																																											

Question		Answer	Marks	Guidance										
2	(iii)	<p>Evidence of trial and improvement towards a solution. E.g.</p> <table border="1"><tr><td>k</td><td>1.1</td><td>1.2</td><td>1.11</td><td>1.109</td></tr><tr><td>Int</td><td>0.9942</td><td>1.0546</td><td>1.0006</td><td>0.99993</td></tr></table> <p>1.11 to 2 dp</p>	k	1.1	1.2	1.11	1.109	Int	0.9942	1.0546	1.0006	0.99993	<p>M1 A1 at least one A1 others A1 answer [4]</p>	
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3	(i)	Euler's method with $h = 0.1$:						M2																																																																																						
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3 (iii)	<p>Minimum points for reducing h:</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th>h</th> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0.1</td> <td>0.7</td> <td>0.731079</td> </tr> <tr> <td>0.05</td> <td>0.75</td> <td>0.729895</td> </tr> <tr> <td>0.025</td> <td>0.725</td> <td>0.729388</td> </tr> <tr> <td>0.0125</td> <td>0.725</td> <td>0.729321</td> </tr> <tr> <td>0.00625</td> <td>0.73125</td> <td>0.729297</td> </tr> </tbody> </table> <p>(0.73, 0.73) seems safe to 2 dp. $x = y$ at the minimum is to be expected because that makes y' zero</p>	h	x	y	0.1	0.7	0.731079	0.05	0.75	0.729895	0.025	0.725	0.729388	0.0125	0.725	0.729321	0.00625	0.73125	0.729297	M1A1 G2 M1A1 A1 E1	New values soi NB: the curve rises slightly if too large a value of h is used.
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Question		Answer	Marks	Guidance																																																
4	(i)	<p>Sketch of data points;</p>  <p>Appears roughly cubic</p>	G2																																																	
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Question			Answer	Marks	Guidance																																			
4	(iv)	(A)	To estimate $f(2)$:																																					
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