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Cambridge Technicals

Engineering

Unit **23:** Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for January 2017

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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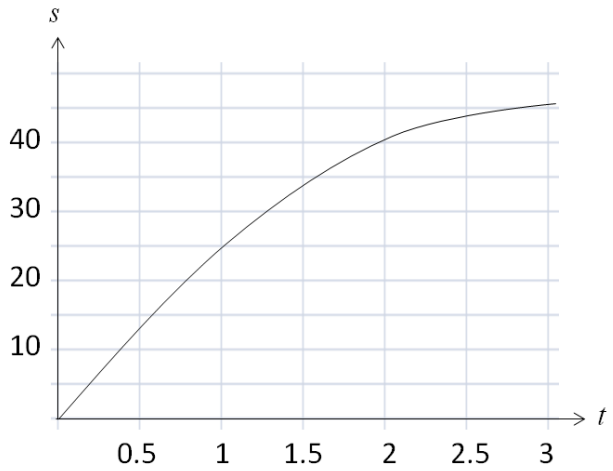
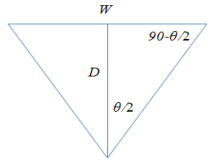
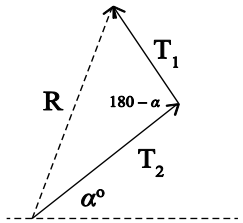
Question			Answer	Marks	Guidance
1	(a)			**B1** **B1** **B1 FT** **[3]**	Suitable scales marked on *s* and *t* axes Plotting of points; allow one error Smooth curve through points plotted; allow FT if correct general shape
	(b)	**(i)**	Using $t = 1$, $25 = A + B$ Using $t = 2$, $40 = 4A + 2B$ $100 = 4A + 4B$ $60 = 2B$ $B = 30$, $A = 25 - 30 = -5$	**B1** **M1** **A1** **[3]**	Two correct equations Eliminate one variable; allow any appropriate method Correct values for *A* and *B*
	(b)	**(ii)**	$30 = -5t^2 + 30t$ $5t^2 - 30t + 30 = 0$ $t^2 - 6t + 6 = 0$ $t = \frac{6 \pm \sqrt{6^2 - 4 \times 1 \times 6}}{2 \times 1} = \frac{6 \pm \sqrt{12}}{2}$ Obtain t= 1.27	**M1** **M1** **A1** **[3]**	Form quadratic using their A and B from (i); Use quadratic formula oe to solve their equation; And no others 1.2679(49…); allow exact equivalent $3 - \sqrt{3}$

Question			Answer	Marks	Guidance
1	(b)	(iii)	u = 30 (ms^{-1})	**B1** **[1]**	From $s = \frac{1}{2}(u+v)t$ with $v = 0$ and $t = 3$; $u = \frac{2\times 45}{3} = 30$ OR using $u = \frac{\mathrm{d}s}{\mathrm{d}t} = 2At + B$; $u(0) = B = 30$ m s^{-1}
1	(b)	(iv)	Deceleration = 10 (ms^{-2})	**B1** **[1]**	Accept $-10ms^{-2}$ From $v^2 = u^2 + 2as$; $0 = 30^2 + 2a\times 45$; $a = \frac{-900}{90} = -10$ OR Compare $s = At^2 + Bt$ with standard formula $s = ut + \frac{1}{2}at^2$; $\frac{1}{2}a = -5$, $a = -10$ m s^{-2} OR $a = \frac{\mathrm{d}u}{\mathrm{d}t} = 2A = 2\times(-5) = -10$ m s^{-2} Accept any correct alternative solutions

Question			Answer	Marks	Guidance
2	(a)		$\tan(\theta/2) = \frac{W/2}{D}$ $W = 2D\tan(\theta/2)$ $A = \frac{2D\tan(\theta/2)D}{2} = D^2\tan(\theta/2)$	M1 A1 A1 [3]	Uses tan = op/adj in right angled triangle Correct expression for width; or half-width if subsequently used appropriately for area AG; necessary detail needed 
2	(b)	(i)	$5d^2\tan(\theta/2)$ [Mass of water displaced = mass of hull] $5d^2\tan(\theta/2)\times 1000 = 4000$ $d^2 = \frac{4}{5\tan(\theta/2)}$ $d = 2\sqrt{\frac{1}{5\tan(\theta/2)}}$ AG	B1 M1 M1 A1 [4]	Correct expression for volume of submerged part of hull soi Form equation; allow $5d^2\tan(\theta/2) = 4$ with **4** justified as volume of displaced water Rearrange to make d^2 the subject of formula AG; sufficient detail needed;
2	(b)	(ii)	$1.7 = 2\sqrt{\frac{1}{5\tan(\theta/2)}}$ $(0.85)^2 = \frac{1}{5\tan(\theta/2)}$ $\tan(\theta/2) = \frac{1}{5\times 0.85^2} = 0.2768(17..)$ $\theta/2 = \tan^{-1}(0.2768) = 15.4729(72..)$ $\theta = 30.9(45944.)$	B1 M1 M1 A1 [4]	Substitution of 1.7 soi Attempt solution by squaring both sides Transpose to make angle the subject of formula Evaluate using inverse tan

Question			Answer	Marks	Guidance
3	(a)		$\overline{T_1} = (T_1 \cos 2\alpha, T_1 \sin 2\alpha)$ $\overline{T_2} = (T_2 \cos \alpha, T_2 \sin \alpha)$	**B1B1** **B1B1** **[4]**	Or $\overline{T_1} = (-T_1 \cos(180 - 2\alpha), T_1 \sin(180 - 2\alpha)$ Two marks available for each tension given in horizontal & vertical component form; other notations acceptable eg a**i**+b**j** and column vectors. SC B2 all 4 components seen separately eg in correct equilibrium statements.
3	(b)			**B1** **B1** **[2]**	3 correct vectors; with directions shown Correct configuration (soi from triangle of forces/parallelogram of forces) with or without angles; no need for R to be vertical
3	(c)		$\overline{R} = (T_1 \cos 2\alpha + T_2 \cos \alpha, T_1 \sin 2\alpha + T_2 \sin \alpha)$	**M1** **A1** **[2]**	Vector addition method for either component Correct answer comprising both components Or $\overline{R} = (-T_1 \cos(180 - 2\alpha) + T_2 \cos \alpha, T_1 \sin(180 - 2\alpha) + T_2 \sin \alpha)$
3	(d)	(i)	$cos2\theta = 2cos^2\theta - 1$ $T_1 \cos 2\alpha + T_2 \cos \alpha = 0$ $T_1(2\cos^2 \alpha - 1) + T_2 \cos \alpha = 0$ $2T_1 \cos^2 \alpha + T_2 \cos \alpha - T_1 = 0$ $\cos \alpha = \frac{-T_2 \pm \sqrt{{T_2}^2 - 4 \times 2T_1 \times (-T_1)}}{4T_1}$	**B1** **M1** **M1** **A1** **[4]**	Correct double angle identity soi For equating their horizontal component to zero Use of appropriate identity and simplification to obtain 3 term quadratic in $cos\alpha$ Obtain expression for cosα; need not be simplified
		(ii)	$\cos \alpha = \frac{-T_1 \pm \sqrt{{T_1}^2 - 4 \times 2T_1 \times (-T_1)}}{4T_1}$ (= 0.5) $\alpha =$ 60°	**M1** **A1** **[2]**	Substitute $T_1 = T_2$ into their answer from (i) and attempt simplification Alternative; equates horizontal components when $T_1 = T_2$ $[T_1] \cos(180 - 2\alpha) = [T_2] cos\alpha$ $180 - 2\alpha = \alpha$; M1 www SC answer only B1

Question			Answer	Marks	Guidance
4	(a)		$\frac{\mathrm{d}\eta}{\mathrm{d}N} = -0.02\left(\frac{N}{100}\right) + \frac{0.06}{10}$ $0.02\left(\frac{N}{100}\right) = \frac{0.06}{10}$ Obtain 30 (rps)	***M1** **DM1** **A1** **[3]**	Differentiate to obtain form $k_1N + k_2$ Dep *M1; equate to zero and attempt to solve www
4	(b)		$\eta = -0.01\left(\frac{30}{10}\right)^2 + 0.06\left(\frac{30}{10}\right) + 0.2 = 0.29$ Obtain ($P = 10000 \times 0.29 \times 30 =$) 87000(W)/87 (kW)	**B1 FT** **B1** **[2]**	Substitute their N to correctly evaluate η ; allow ECF from (a) From 30 obtained in (a)
4	(c)		$P = 10000\left(-0.01\left(\frac{N}{10}\right)^2 + 0.06\left(\frac{N}{10}\right) + 0.2\right)N$ $= 10000\left(-0.01\frac{N^3}{100} + 0.06\frac{N^2}{10} + 0.2N\right)$ $= -N^3 + 60N^2 + 2000N$ $\frac{\mathrm{d}P}{\mathrm{d}N} = -3N^2 + 120N + 2000 = 0$ $N = \frac{-120 \pm \sqrt{120^2 - 4 \times (-3) \times 2000}}{-6}$ Obtain N= 52.6598(6...)	**B1** **M1** **M1** **M1** **A1** **[5]**	Correct equation for P; soi Simplify to obtain a cubic polynomial in N Differentiate to obtain a quadratic and equate to zero Solve a 3-term quadratic
4	(d)		Compare earlier answers to draw appropriate conclusion The claim is NOT true.	**M1** **A1** **[2]**	Alternatives; eg calculates power at speed corresponding to their maximum efficiency CAO; reference to the student's claim

Question			Answer	Marks	Guidance
5	(a)		$\frac{dP}{dh} = 4(-\frac{he^{-\frac{h}{2}}}{2} + e^{-\frac{h}{2}})$	*M1	Differentiate using product rule to obtain two terms
				A1	Correct derivative
			$= 4e^{-\frac{h}{2}}(1-\frac{h}{2}) = 0$	DM1	Dep *M1; equate to zero and attempt to solve
			$\Rightarrow h = 2$	A1	www; and no other solutions
				[4]	
5	(b)		Identifies u and v' correctly	M1	Correct direction chosen for integration by parts; soi
			$\int e^{-\frac{h}{2}} = -2e^{-\frac{h}{2}}$	B1	
			Constructs correct form for integration by parts	M1	Integration by parts as far as $f(h) +/- \int g(h)dx$
			Obtains $\frac{4}{10}\left\{-2he^{-\frac{h}{2}} - \int -2e^{-\frac{h}{2}}\right\}_0^{10}$	A1	Condone absence of limits here
			$= \frac{4}{10}\left\{-2he^{-\frac{h}{2}} - 4e^{-\frac{h}{2}}\right\}_0^{10}$	M1	Attempt to integrate their remaining term
			$= \frac{4}{10}\left\{(-20e^{-5} - 4e^{-5}) - (-4)\right\}$	M1	Substitution of limits soi
			$\frac{4}{10}\left\{-24e^{-5} + 4\right\} = 1.535(316.)$ (kW)	A1	
				[7]	

Question			Answer	Marks	Guidance
6	(a)				Accept correct alternative workings throughout
			$\frac{1}{Z_t} = \frac{1}{R} + \frac{1}{\omega L\text{j}} + \omega C\text{j}$	B1	Insert component impedances
			use of $\text{j}^2 = -1$	B1	Soi anywhere in solution
			$= \frac{1}{R} - \frac{j}{\omega L} + \omega C\text{j} \;= \frac{\omega L - R\text{j} + \omega^2 RCL\text{j}}{R\omega L}$	M1	Common denominator; denominator may still include j $\frac{\omega Lj + R + R\omega^2 Lcj^2}{R\omega Lj}$
			$= \frac{\omega L + R(\omega^2 RCL - 1)\text{j}}{R\omega L}$	DM1	Simplification with $R\omega L$ as denominator; dep on all earlier marks
			$Z_t = \frac{R\omega L}{\omega L + R(\omega^2 LC - 1)\text{j}}$ AG	A1	Invert to obtain given answer; CAO; sufficient detail needed
				[5]	
6	(b)	(i)	$Z_t = \frac{1\times 10^4 \times 10^{-2}}{10^4 10^{-2} + 1(10^8 10^{-2} 10^{-5} - 1)\text{j}} = \frac{100}{100 + 9j}$	B1	Insert values and simplify; accept 10^2
			$= \frac{100(100 - 9\text{j})}{10000 + 81}$	M1	Use of complex conjugate to rationalise expression;
			$= \frac{10000 - 900\text{j}}{10081}$		
			$= 0.9919(65..) - \text{j}0.08927(68..)$	A1	Accept decimals or fractions
				[3]	

Question			Answer	Marks	Guidance
6	(b)	(ii)	$r \approx \sqrt{0.992^2 + 0.0893^2}$ $= \sqrt{0.992^2 + 0.0893^2}$ $= 0.99(6..)$ $\theta \approx \tan^{-1}\left(\frac{-0.893}{0.992}\right)$ $= -5.1(4..)^o$	**M1** **A1** **M1** **A1** **[4]**	Use of correct formula for r; soi Evaluation of r Appropriate use of inverse tan $(\pm)$; soi Award full marks for correct values of r and θ seen.
7	(a)		$\frac{(1+x)-x(1)}{(1+x)^2}$ $= \frac{1}{(1+x)^2}$ AG	**M1** **A1** **[2]**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{1+x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{vu'-uv'}{v^2}$ Application of quotient formula; product rule alternative AG; sufficient detail needed

Question			Answer	Marks	Guidance
7	(b)		$\frac{d\omega}{dt} = \omega_0\left(\frac{t}{(1+t)^2} - \frac{1}{(1+t)}\right)$	**M1**	Division by $(1 + t)$ to separate variables; accept unsimplified
			$= \omega_0\left(\frac{t}{(1+t)^2} - \frac{(1+t)}{(1+t)^2}\right)$ $= \omega_0\left(-\frac{1}{(1+t)^2}\right)$	**A1**	Simplify to obtain single fraction
			$\int 1 d\omega = -\omega_0 \int \frac{1}{(1+t)^2} dt$		Integrate (both sides); condone absence of $+c$
			$\omega = -\omega_0\left(\frac{t}{1+t}\right) + c$	**M1**	Apply initial conditions to evaluate c ; oe from use of limits
			$\omega = \omega_0$ when $t = 0 \Rightarrow c = \omega_0$	**M1**	CAO
			$\omega = \omega_o\left(1 - \frac{t}{1+t}\right)$ AG	**A1**	
				[5]	
7	(c)		Speed decreases with time as expected (so realistic) The flywheel never completely stops (so not realistic)	**B1** **B1** **[2]**	Soi Accept any 2 reasonable observations

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