OCR
Oxford Cambridge and RSA

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873

Unit 23: Applied mathematics for engineering

Monday 5 June 2017 – Afternoon

Time allowed: 2 hours

You must have:
- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name		Last Name	
Centre Number		Candidate Number	
Date of Birth	D D M M Y Y Y Y		

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **24** pages

FOR EXAMINER USE ONLY	
Question No	Mark
1	/12
2	/13
3	/12
4	/12
5	/12
6	/11
7	/8
Total	**/80**

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Question 1 begins on page 3

Answer **all** questions.

1 Fig.1 shows a sector of a circle with centre O and radius r. The angle of the sector is θ radians.

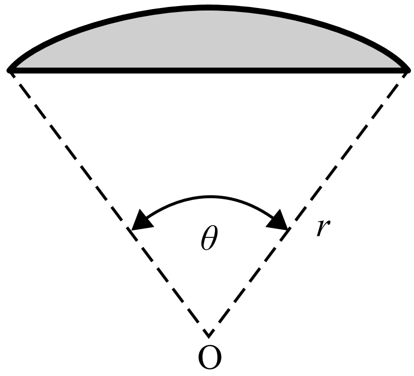

Fig. 1

(a) Show that the area, A, of the segment of the circle shown shaded is given by

$$A = \frac{r^2(\theta - \sin\theta)}{2}.$$

[3]

(b) Fig. 2 shows the face of a steel bush which has a uniform cross section. The thickness of the bush is 12 mm as shown in Fig. 3.

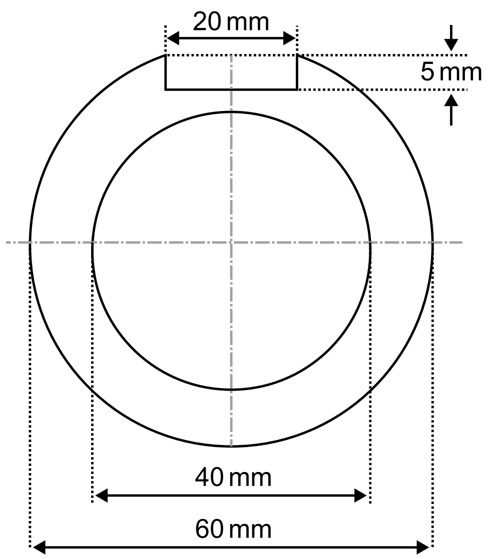

Fig. 2

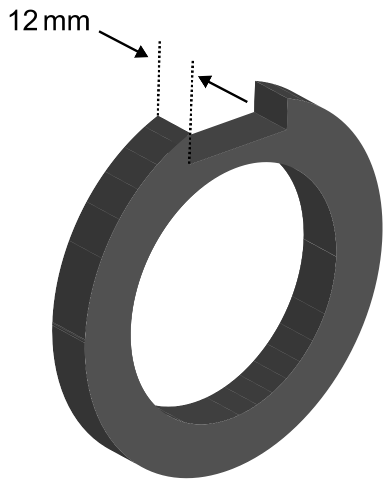

Fig. 3

(i) Calculate the surface area of the face of the bush giving your answer to the nearest whole number. **[6]**

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(ii) Calculate the volume of the bush.

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(iii) The density of steel used for making the bush is 8000 kg m^{-3}.

Calculate the mass of the bush giving your answer in grams.

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2 Fig. 4 show a circuit diagram containing three resistors with resistances R_1 kΩ, R_2 kΩ and R_3 kΩ. Currents flowing in the circuit are i_A, i_B, i_1, i_2 and i_3 as shown on Fig.4; these are measured in mA.

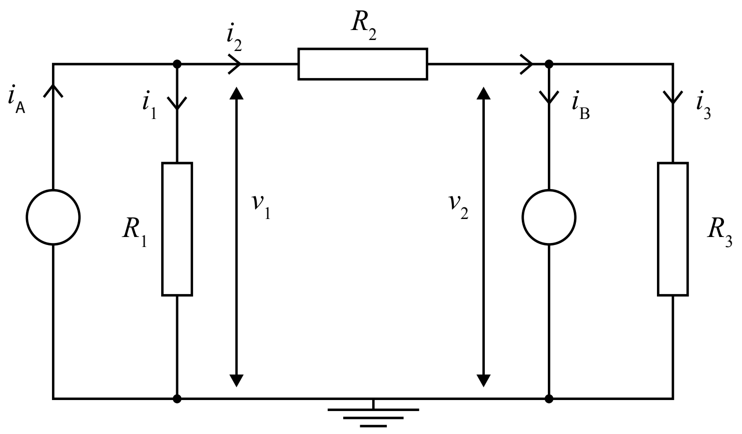

Fig. 4

The following values are known.

$R_1 = 12$ $\quad R_2 = 9$ $\quad R_3 = 6$

$i_A = 1$ $\quad i_B = 4$

The voltage levels v_1 and v_2 as indicated on Fig.4 are to be found.

Using Kirchhoff's current law the following equations can be established.

$$i_1 + i_2 = i_A \qquad i_2 - i_3 = i_B$$

Using Ohm's law the following equations can be established.

$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_2}{R_3}$$

(i) Using the values and equations above, show that v_1 and v_2 satisfy the following equations.

$$7v_1 - 4v_2 = 36$$
$$2v_1 - 5v_2 = 72$$

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(ii) Express these simultaneous equations in matrix notation.

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(iii) Using matrix algebra calculate the values of v_1 and v_2.

[6]

3 Fig. 5 shows a simple hydraulic lifting mechanism consisting of two cylinders, C_1 and C_2, and a connecting tube. The cylinders contain pistons, P_1 and P_2, with diameters 100 mm and 200 mm respectively. The pistons are in direct contact with a non-compressible hydraulic fluid, of density ρ, which fills the lower part of each cylinder and the connecting tube. When P_1 is forced downwards the pressure in the liquid increases and this causes P_2 and the load P_2 supports to be forced upwards.

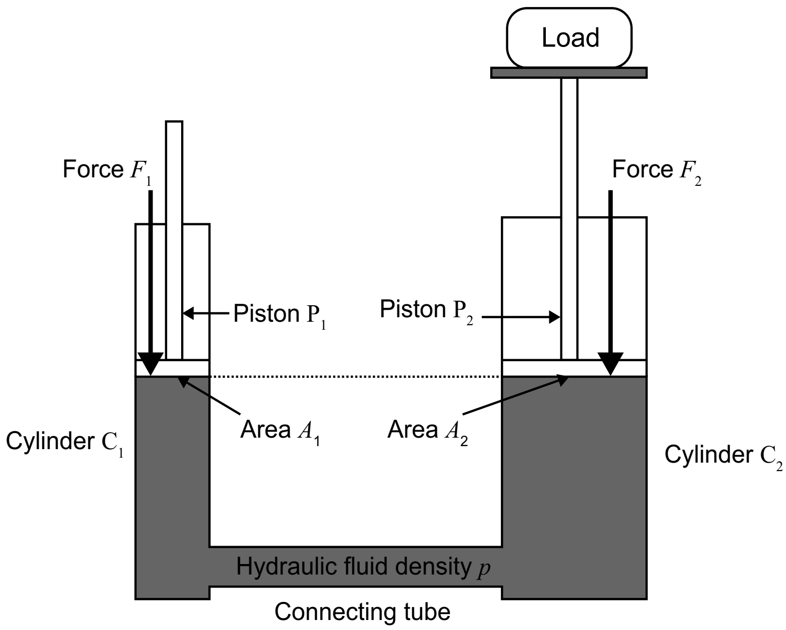

Fig. 5

(i) Given that the volume of the hydraulic fluid remains constant, show that when piston P_1 is moved down a distance of Δd piston P_2 will move up a distance of $\Delta h = \frac{\Delta d}{4}$.

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(ii) The mass of P_2 together with its load is 100 kg; this total mass provides a downward force of F_2 N acting on the fluid at P_2. At P_1 a downward force of F_1 N acts on the fluid. For this part of the question you may assume that, to maintain the pistons in equilibrium at the same horizontal level,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2},$$

where A_1 m^2 and A_2 m^2 are the areas of P_1 and P_2 respectively which are in direct contact with the fluid.

Calculate the force, F_1 N required to maintain the two pistons at the same horizontal level.

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(iii) If P_2 is to be held in equilibrium at a height of h m above the height of P_1, as shown in Fig. 6, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} + \rho g h$$

where $\rho = 860\,\text{kg m}^{-3}$,
$g = 9.8\,\text{m s}^{-2}$.

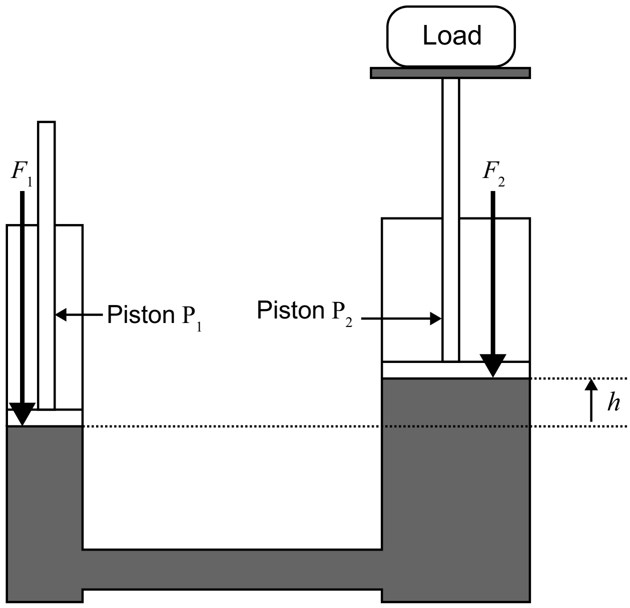

Fig. 6

The two pistons are initially at the same level, as shown in Fig. 5 on page 9. The force F_1 is increased to 350 N, and subsequently P_2 is held in equilibrium h m above the level of P_1, as shown in Fig. 6. Calculate the **height gained**, Δh, by P_2 and its load.

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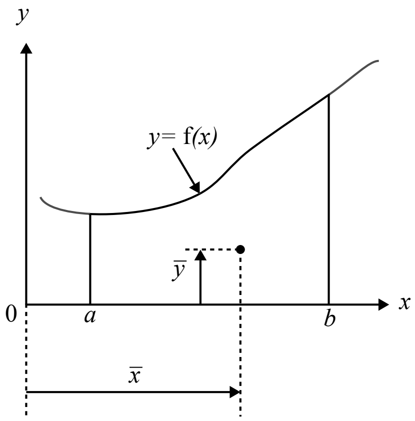
4 For this question you should use the following formulae which give the coordinates, $\overline{x}$ and $\overline{y}$, of the centroid of a plane figure bounded by the curve with equation $y = \mathrm{f}(x)$, the x-axis and the lines $x = a$ and $x = b$ as shown in Fig. 7.



$$\overline{x} = \frac{\int_a^b xy\,dx}{\int_a^b y\,dx}$$

$$\overline{y} = \frac{\int_a^b \tfrac{1}{2}y^2\,dx}{\int_a^b y\,dx}$$

Fig. 7

A metal plate is formed from the region bounded by the curve with equation $y = x^2 + 1$, the x- and y-axes and the line $x = 2$, as shown in Fig. 8.

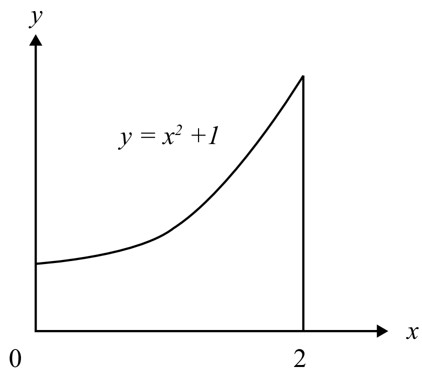


Fig. 8

(i) Calculate the area of the metal plate.

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(ii) Calculate $\overline{x}$.

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(iii) Calculate $\overline{y}$.

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5 For this question you should use the following result.

$$\text{If } y = e^{-\left(\frac{x}{a}\right)^2} \text{ then } \frac{dy}{dx} = -\frac{2x}{a^2}e^{-\left(\frac{x}{a}\right)^2}.$$

An artificial ski slope is modelled by the equation

$$h = 30e^{-\left(\frac{x}{40}\right)^2},$$

where h m is the height of the slope above horizontal ground level and x m is the horizontal distance measured from the start of the slope.

(i) Sketch the graph of h against x for $0 \le x \le 80$.

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(ii) Calculate x when $h = 15$.

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(iii) Calculate the height of the slope at the point where the **gradient** is a maximum.

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6 For this question you are given the following equations for a uniform circular object (such as a sphere or disc) rolling down a slope with a linear velocity of v m s^{-1} and rotating, without slipping, at an angular speed of ω rad s^{-1} . The object has a mass of m kg, a radius of r m and is positioned at a height of h m above a fixed level.

Relative gravitational potential energy $W_P = mgh$

Linear kinetic energy $W_K = \frac{1}{2}mv^2$

Rotational kinetic energy $W_R = \frac{1}{2}I\,\omega^2$

where moment of inertia $I = mcr^2$ and

for a solid sphere $c = \frac{2}{5}$ or for a solid disc $c = \frac{1}{2}$.

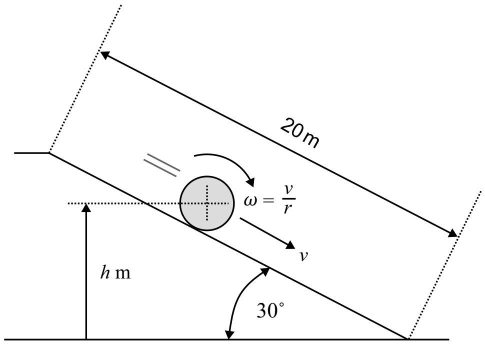

Fig. 9

(i) A circular object with radius r m and mass m kg is released from rest at the top of a slope. The slope is inclined at a constant angle of 30° to the horizontal. The object rolls down the slope without slipping so that both its linear and rotational velocity increase with constant acceleration, as shown in Fig. 9.

You may assume that total energy, W_T J, remains constant during its travel and is given by $W_T = W_P + W_K + W_R$.

Show that the linear speed of the object when it has travelled 20 m down the slope is given by

$$v = \sqrt{\frac{20g}{1+c}}.$$

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(ii) A solid sphere and a solid disc are released from rest at the top of the slope. Use the formula for v given in part (i) and an appropriate constant acceleration formula to calculate the time it will take for each object to travel a distance of 20 m down the slope.

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7 For this question you may use $\int \frac{1}{a+bx}\,\mathrm{d}x = \frac{\ln|a+bx|}{b} + c$.

Fig. 10 shows an electronic circuit containing a resistor with resistance $R\,\Omega$, an inductor with inductance L H, and a DC voltage supply of v V. When the switch is in the open position, as shown in Fig. 10, the current, i A, flowing through the circuit is zero.

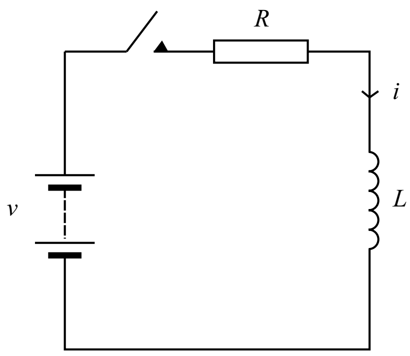

Fig. 10

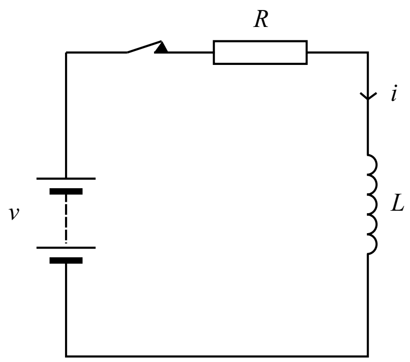

Fig. 11

At time $t = 0$ s the switch is closed as shown in Fig. 11; the current now flowing through the circuit can be modelled by the following differential equation.

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri = v$$

Solve the differential equation to find an expression for the current, i, flowing though the circuit. Give your answer in terms of R, L, v and t. **[8]**

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END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional answer space is required, you should use the following lined pages. The question number(s) must be clearly shown – for example 1(d) or 6(b).