Cambridge Technicals

Engineering

Unit **23:** Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Subject-specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.
Mark in using a red pen.
Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.
Transfer these marks to the box on the front page.
Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM
A method mark which is dependent on a previous method mark.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless

the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

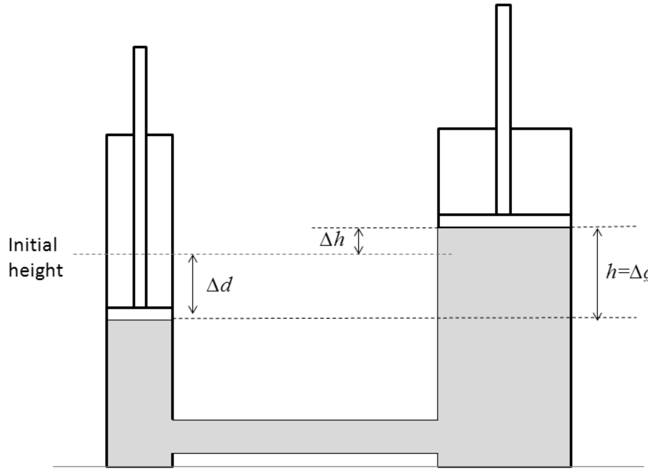
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	a		Area of sector = $\dfrac{r^2\theta}{2}$ Area of (isosceles) triangle = $\dfrac{r^2\sin\theta}{2}$ Area of shaded region= $\dfrac{r^2\theta}{2}-\dfrac{r^2\sin\theta}{2}=\dfrac{r^2(\theta-\sin\theta)}{2}$ AG	M1 M1 A1 [3]	Subtraction of terms must be seen for A1
1	b	i	Sight of 20×5 $\pi\left(\dfrac{60}{2}\right)^2-\pi\left(\dfrac{40}{2}\right)^2$ $500\pi-20\times5$ $=1470(.796)$ $\dfrac{\theta}{2}=\sin^{-1}\dfrac{1}{3}=0.3398(369..)$ Area of face = $\pi30^2-\pi20^2-20\times5-30^2\dfrac{(0.6796-0.6285)}{2}=(1447.8)$ $\approx1448\ (\text{mm}^2)$	B1 M1 M1 A1 M1 A1 [6]	Annulus area soi (dimensions may be mm/cm/m) Rectangular portion removed from *their attempt* at annulus area Note: First 4 marks can be earned if embedded in attempt at the total surface area of *all* faces of the bush. Calculation of half angle in radians or degrees (19.47°) $\theta=0.6796(737..)$ $\sin\theta=0.6285(396..)$ Cao from complete method for area of face, incorporating segment area from part (i)
1	b	ii	volume = $1448\times12=17376\ (\text{mm}^3)$	B1√ [1]	*Their area* × 12 as FT or fresh start Note: 1471 leads to 17649.6 3sf acceptable
1	b	iii	mass = $17376\times10^{-9}\times8000\times10^3$ ≈ 139 (g) CAO	M1 A1 [2]	FT *their volume* × 8000 (Note: Use of 1471leads to 141.2g)

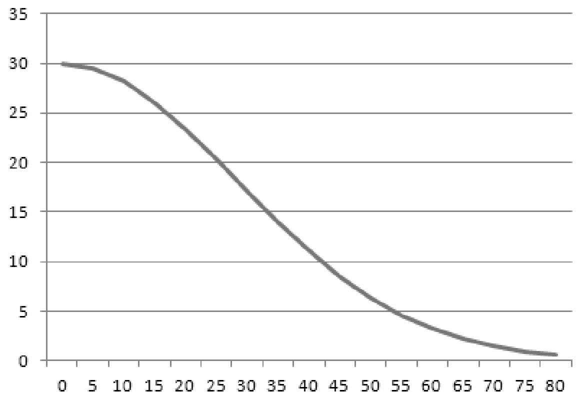
Question			Answer	Marks	Guidance
2	i				Notes: accept working in other consistent units eg with amps rather than mA and ohms rather than $k\Omega$. Follow workings presented for method marks. Resistances may be substituted straight away. Acceptable to form initial equations that include i_1, i_2, i_3 terms.
			$\frac{v_1}{R_1}+\frac{v_1-v_2}{R_2}=1$	M1	Addition of terms soi and equated to 1
			$\frac{v_1-v_2}{R_2}-\frac{v_2}{R_3}=4$	M1	Subtraction of terms soi and equated to 4
				M1	Both resistances substituted to obtain two distinct equations. Equations may still include current (i) terms at this point
			$\frac{v_1}{12}+\frac{v_1-v_2}{9}=1$ and $\frac{v_1-v_2}{9}-\frac{v_2}{6}=4$ oe	A1	Two correct equations given in terms of v_1 and v_2 only.
			$\Rightarrow 7v_1-4v_2=36$ AG $\Rightarrow 2v_1-5v_2=72$ AG	A1 [5]	Given equations obtained legitimately, with sufficient intermediate steps shown
	ii		$\begin{bmatrix}7 & -4\\2 & -5\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}36\\72\end{bmatrix}$	B2 [2]	$\{Av=c\}$ B1 for $\begin{bmatrix}7 & -4\\2 & -5\end{bmatrix}$ seen

Question			Answer	Marks	Guidance
	iii		$\begin{bmatrix}7 & -4\\2 & -5\end{bmatrix}^{-1} = \frac{1}{\det(A)}\begin{bmatrix}-5 & 4\\-2 & 7\end{bmatrix}$	**M1**	
			$\det(A) = -27$	**A1**	From $7\times(-5)-(-4)\times 2$
			$\begin{bmatrix}v_1\\v_2\end{bmatrix} = \begin{bmatrix}7 & -4\\2 & -5\end{bmatrix}^{-1} \times \begin{bmatrix}36\\72\end{bmatrix}$	**M1**	$\{v=A^{-1}c\}$ Correct use of inverse matrix soi
			$\begin{bmatrix}v_1\\v_2\end{bmatrix} = -\frac{1}{27}\begin{bmatrix}-5 & 4\\-2 & 7\end{bmatrix} \times \begin{bmatrix}36\\72\end{bmatrix}$	**A1√**	FT on *determinant* only. May be simplified eg $-\frac{36}{27}\begin{bmatrix}-5 & 4\\-2 & 7\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$ oe
			$v_1 = -\frac{36}{27}(-5\times1+4\times2) = -4$	**M1**	For matrix × vector seen once
			$v_2 = -\frac{4}{27}(-2\times1+7\times2) = -16$	**A1**	For both results
				[6]	

Question			Answer	Marks	Guidance
3	i		Volume in cylinder C_2 is increased by $\pi(100)^2 \times \Delta h$ Volume in cylinder C_1 is decreased by $\pi(50)^2 \times \Delta d$	**M1**	Both required soi. Expect $10000\pi\ \Delta h$ and $2500\pi\ \Delta d$ oe with consistent units
			$\pi(50)^2 \times \Delta d = \pi(100)^2 \times \Delta \text{h}$	**M1**	Volumes equated oe
			$\Delta d = 4\Delta h$	**M1**	Simplification steps may be implied
			$\Delta h = \frac{\Delta d}{4}$ AG	**A1**	Cao using given symbols
				[4]	ALT: Linear SF = $^1/_2$ B1. Area SF = $^1/_4$ B1. Sets up appropriate equation M1 and obtains AG A1. SC2: Use of πD^2 rather than πr^2 leading to given answer B2
3	ii				Areas may be calculated: $A_1 = \pi\left(\frac{0.1}{2}\right)^2$ and $A_2 = \pi\left(\frac{0.2}{2}\right)^2$ oe
			$F_2 = mg = 100 \times 9.8 = 980\,\text{N}$	**B1**	
			$F_1 = \frac{1}{4} \times F_2$	**M1**	Soi
			$= 245$ (N)	**A1** **[3]**	

Question			Answer	Marks	Guidance
3	iii		$h = \dfrac{\left(\dfrac{F_1}{A_1} - \dfrac{F_2}{A_2}\right)}{\rho g}$	M1	Formula transposed at some point, with or without values substituted
			$h = \dfrac{\left(\dfrac{350}{\pi(0.05)^2} - \dfrac{980}{\pi(0.1)^2}\right)}{860 \times 9.8}$	M1	Substitution of correct values required (but terms may still involve g)
			$= 1.58(626)\ m$	A1	
			But $h = \Delta d + \Delta h = 4\Delta h + \Delta h = 5\Delta h$	DM1	Dependent on M1M1
			$\Delta h = \dfrac{1.5863}{5} = 0.3172(5...)\ \text{m}$	A1	
				[5]	

Question			Answer	Marks	Guidance
4	i		Area = $\int_0^2 (x^2 + 1)\mathrm{d}x$	**M1**	
			$\left[\frac{x^3}{3} + x\right]_0^2 =$	**A1**	Condone missing limits
			$\frac{8}{3} + 2 = \frac{14}{3}$	**A1**	Accept 4.67
				[3]	
4	ii		$\int_0^2 (x^3 + x)\mathrm{d}x$	**M1**	Integral formed and attempted
			$= \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^2$	**A1**	Condone missing limits
			$6 \div \frac{14}{3}$	**M1**	*Their* evaluated integral ÷ *their* part (i)
			$= \frac{9}{7}$	**A1**	Accept exact equivalents and 1.29
				[4]	

Question			Answer	Marks	Guidance
4	iii		$\int_0^2 \frac{1}{2}(x^2+1)^2\,dx$	**M1**	Integral formed
			$= \frac{1}{2}\int_0^2 (x^4+2x^2+1)\ \ dx$	**A1**	Correct expansion
			$= \frac{1}{2}\left[\frac{x^5}{5}+\frac{2}{3}x^3+x\right]_0^2$	**A1**	Condone missing limits
			$\frac{103}{15} \div \frac{14}{3}$	**M1**	*Their* evaluated integral ÷ *their* part (i)
			$= \frac{103}{70}$	**A1**	Accept exact equivalents and 1.47
				[5]	
5	i			**B1**	Starts at $h = 30$ (condone non-zero gradient at $x = 0$) Value 30 must be shown
				B1	Continuous downward trend, must pass through all points if plotted
				B1	Approaches 0 as $x \Rightarrow 80$ Value 80 must be shown
				[3]	

Question			Answer	Marks	Guidance
5	ii		$30e^{-\left(\frac{x}{40}\right)^2}=15$	M1	
			$e^{-\left(\frac{x}{40}\right)^2}=0.5$	M1	
			$-\left(\frac{x}{40}\right)^2=\ln 0.5$	A1	Or correct equivalent
			$x=33.3$	A1 [4]	
5	iii		$\frac{\mathrm{d}h}{\mathrm{d}x}=30\left(-\frac{2x}{40^2}e^{-\left(\frac{x}{40}\right)^2}\right)$	B1	Note: form of derivative is given in question
				M1	Use of product rule
			$\frac{\mathrm{d}^2h}{\mathrm{d}x^2}=-\frac{60}{40^2}\left(x\left(-\frac{2x}{40^2}\right)e^{-\left(\frac{x}{40}\right)^2}+e^{-\left(\frac{x}{40}\right)^2}\right)$	A1	
			Equate second derivative to zero and solve for x	DM1	Dependent on M1. Expect $x^2=800$
			$h=30e^{-\left(\frac{\sqrt{800}}{40}\right)^2}=30e^{-\frac{800}{1600}}=30e^{-0.5}=18.1959(1...)$	A1 [5]	Accept 18.2 or better

Question			Answer	Marks	Guidance
6	i		Initial energy $W_P = mg \times 20\sin(30)$	M1	Expect 10 mg
			Final energy $W_K + W_R = \frac{1}{2}mv^2 + \frac{1}{2}cmv^2$	M1	Rotational energy could still be in terms of angular speed
			$10mg = \frac{1}{2}mv^2 + \frac{1}{2}cmv^2$	A1	Equate initial energy with final energy; This equation or equivalent with ω term present must be seen
			$20g = v^2 + cv^2$	M1	Simplification by removal of m; terms in linear speed only now
			$v = \sqrt{\frac{20g}{1+c}}$ AG	A1 [5]	Obtained www and sight of $v^2(1+c)$

Question			Answer	Marks	Guidance
6	ii			**M1**	Substitute value of c for either sphere or disc
			For sphere $v=\sqrt{\dfrac{20g}{1+\frac{2}{5}}}=11.83..$	**A1**	Evaluated correctly
			For disc $v=\sqrt{\dfrac{20g}{1+\frac{1}{2}}}=11.43..$	**A1**	Evaluated correctly
			$20=\frac{1}{2}(0+v)t \rightarrow t=\frac{40}{v}$ oe	**M1**	Correct use of $s=\frac{1}{2}(u+v)t$ to obtain expression for time
			Sphere: $t=\frac{40}{11.83}=3.38$	**M1**	Sphere and disc both required: $40 \div$ *their* v
			Disc: $t=\frac{40}{11.43}=3.50$	**A1** **[6]**	Both correct to 2dp

Question			Answer	Marks	Guidance
7			$L\dfrac{\mathrm{d}i}{\mathrm{d}t}=v-Ri$	**B1**	Rearrange
			$\int \dfrac{L}{v-Ri}\mathrm{d}i=\int 1\,\mathrm{d}t$	**B1**	Separate variables ready for integration
				M1	Integrate both sides to obtain correct form (minus omitted from denominator is M1A0)
			$\dfrac{L\ln(v-Ri)}{-R}=t+c_1$	**A1**	oe: Condone absence of c here, but not subsequently. Condone repeated use of c to represent different arbitrary constants
			$v-Ri=e^{-\frac{Rt}{L}+c_2}$	**M1**	Rearrange and take anti logs
			$Ri=v-c_3e^{-\frac{Rt}{L}} \rightarrow i=\dfrac{v}{R}-c_4e^{-\frac{Rt}{L}}$	**M1**	Rearrange for *i*: no new errors
			$i=0$ when $t=0 \Rightarrow c_4=\dfrac{v}{R}$	**M1**	Use initial condition at appropriate point
			$i=\dfrac{v}{R}(1-e^{-\frac{Rt}{L}})$	**A1** **[8]**	substitute $c_4=\dfrac{v}{R}$ for final answer

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