

**OCR**

Oxford Cambridge and RSA

# Level 3 Cambridge Technical in Engineering

## 05823/05824/05825/05873

### Unit 23: Applied mathematics for engineering

**Wednesday 17 January 2018 – Afternoon**

**Time allowed: 2 hours**

**You must have:**

- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name						Last Name				
Centre Number						Candidate Number				
Date of Birth	D	D	M	M	Y	Y	Y	Y		

### INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

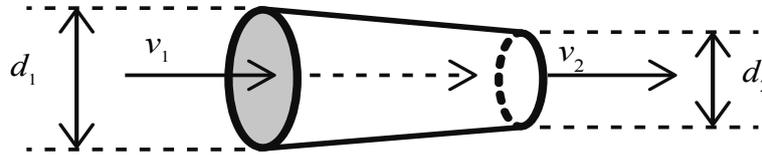
### INFORMATION

- The total mark for this paper is **80**.
- The marks for each question are shown in brackets [ ].
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **20** pages.

FOR EXAMINER USE ONLY	
Question No	Mark
1	/10
2	/13
3	/11
4	/11
5	/12
6	/11
7	/12
<b>Total</b>	<b>/80</b>

Answer **all** the questions.

- 1 Fig. 1 shows part of a circular pipe. The diameter of the pipe varies. At a part of the pipe where the diameter is  $d_1$  m the water is flowing at a rate of  $v_1$  m s<sup>-1</sup>. At another part, where the diameter is  $d_2$  m, the water is flowing at a rate of  $v_2$  m s<sup>-1</sup>.



**Fig. 1**

The volume flow rate,  $V$  m<sup>3</sup> s<sup>-1</sup>, of water flowing at a rate of  $v$  m s<sup>-1</sup> through a section of the pipe with diameter  $d$  m is given by

$$V = v\pi\left(\frac{d}{2}\right)^2.$$

- (i) When the water volume flow rate remains constant along the pipe shown in Fig. 1 show that

$$v_2 = v_1\left(\frac{d_1}{d_2}\right)^2.$$

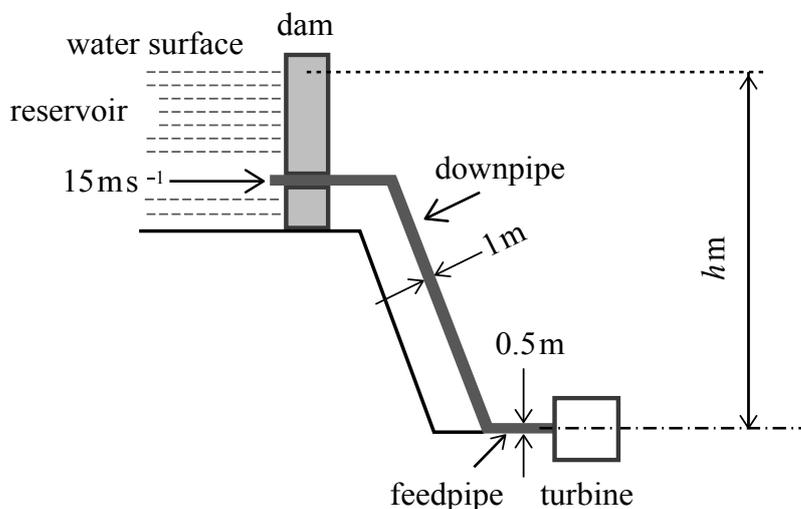
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Fig. 2 shows a dam constructed at the end of a reservoir. Water flows from the dam through a circular downpipe of diameter 1 m. The downpipe is connected to a circular feedpipe of diameter 0.5 m which passes water into the turbine. The surface of the water in the reservoir is  $h$  m above the turbine.



**Fig. 2**

- (ii) Water flows from the dam through the downpipe at the rate of  $15 \text{ m s}^{-1}$ . Calculate the rate,  $v \text{ m s}^{-1}$ , at which the water flows through the feedpipe and enters the turbine.

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- (iii) For this system the rate at which water enters the turbine,  $v \text{ m s}^{-1}$ , is also given by the formula

$$v = \sqrt{2gh}.$$

Using the value of  $v$  found in part (ii) calculate the value of  $h$ .

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- (iv) When the volume flow rate of water entering the turbine is  $V \text{ m}^3 \text{ s}^{-1}$  the corresponding mass flow rate is  $Q \text{ kg s}^{-1}$ , where  $Q = 1000V$ .

Calculate the values of  $V$  and  $Q$ .

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- (v) The power,  $P \text{ W}$ , produced by the turbine is given by

$$P = \eta Qgh, \text{ where } \eta \text{ is the efficiency of the turbine.}$$

Calculate the efficiency of the turbine when the power produced is  $20 \text{ MW}$ .

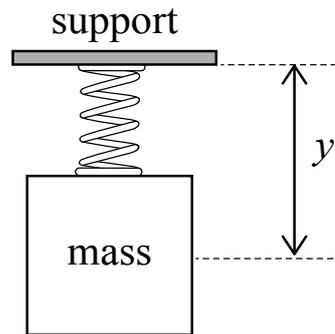
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**Question 2 begins on page 5**

- 2 Fig. 3 shows a mass supported by an energised coil spring which is fixed securely to an overhead support.



**Fig. 3**

As the spring expands and contracts the distance  $y$  mm from the support to the centre of the mass oscillates according to the formula

$$y = 60 + 20 \sin(4\pi t),$$

where  $t$  is time measured in seconds.

- (i) State the technical name for oscillating movement of this type.

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- (ii) Calculate

- (A) the frequency of the oscillation in cycles per second,

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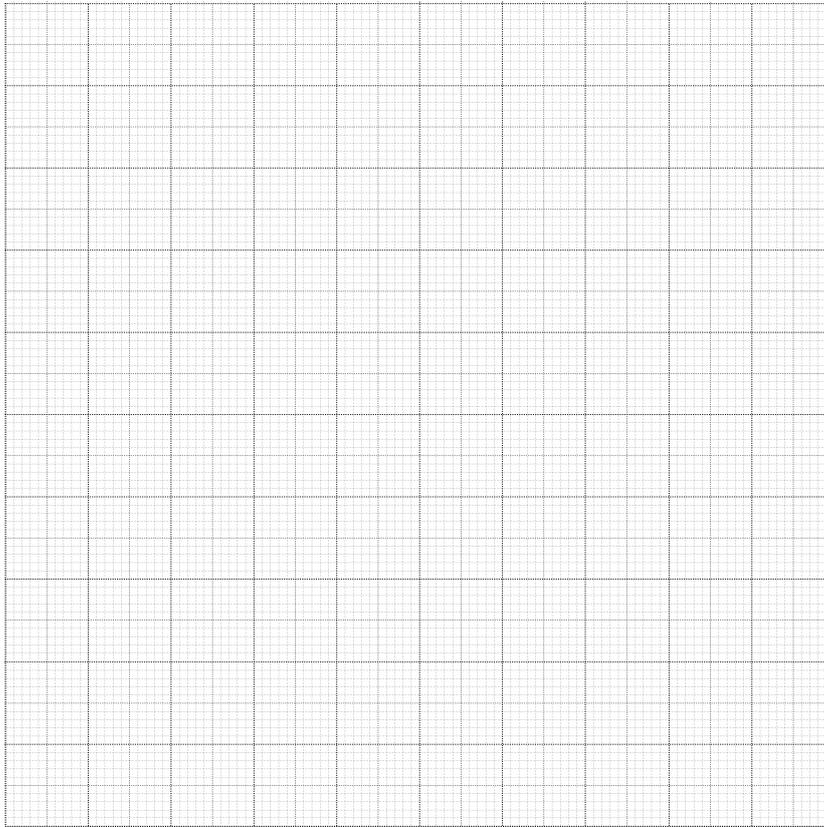
- (B) the period of the oscillation in seconds.

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 ..... [1]

(iii) On the grid below, sketch the graph of

$$y = 60 + 20 \sin(4\pi t) \text{ for } 0 \leq t \leq 1.$$

Label the axes.



[3]

(iv) In practice the spring loses energy as it expands and contracts. In this case the distance  $y$  mm is given by

$$y = 60 + 20 e^{-t} \sin(4\pi t).$$

(A) The table below shows some values for  $t$  and  $y$ . Work out the missing  $y$ -values and complete the table.

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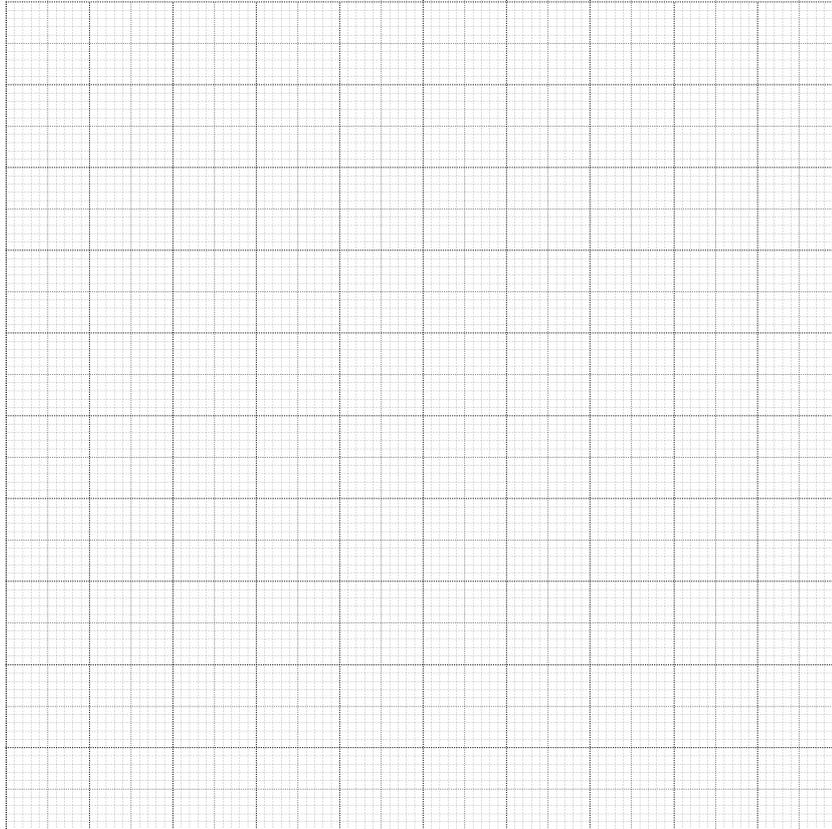
$t$	0.125	0.375	0.625	0.875
$y$	77.65			

[3]

(B) Plot these values on the grid below and draw the graph of

$$y = 60 + 20 e^{-t} \sin(4\pi t) \text{ for } 0 \leq t \leq 1.$$

Label the axes.



[2]

(C) Use your graph in part (iv)(B) to estimate the values of  $t$  when  $y = 50$ .

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..... [2]



- (ii) An engineer has estimated the volume of steel in the tube by multiplying the outside surface area by the wall thickness. Explain why this calculation does not produce an accurate result and, without performing any additional calculations, describe a method that will produce an accurate value.

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- 4 Two vertical poles, each of height 10 m, are sited on the side of a hill. The horizontal distance between the poles is 30 m and a cable is suspended between them. Fig. 6 shows the two poles, the cable and the side of the hill beneath the cable aligned within a Cartesian coordinate system  $(x, y)$ . The base of the pole on the left is at the origin  $(0, 0)$ .

The cable may be modelled by  $y = 0.01x^2 - 0.4x + 10$ .

The side of the hill may be modelled by  $y = -0.001x^2 - 0.06x$ .

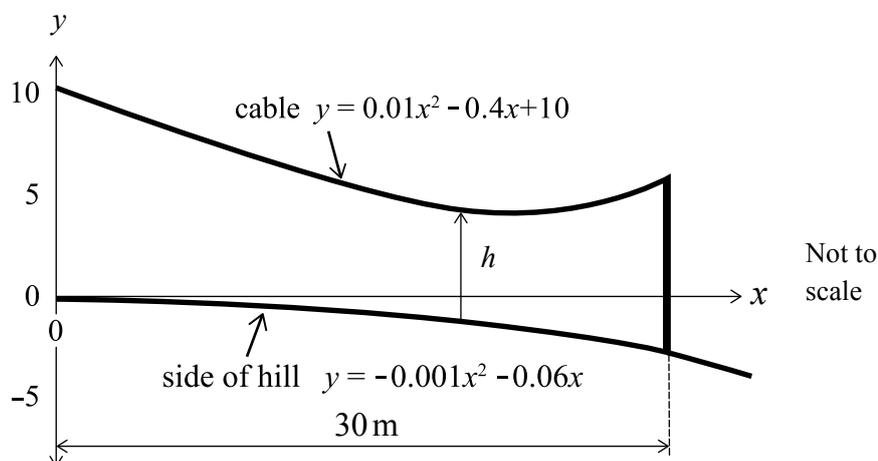


Fig. 6

- (i) Find an equation,  $h = F(x)$ , where  $F(x)$  defines the vertical height of the cable above the side of the hill for  $0 \leq x \leq 30$ .

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- (ii) Calculate the minimum height of the cable above the side of the hill. You must show that your answer is a minimum.

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- 5 Fig. 7 shows a schematic diagram of a crank and piston mechanism.

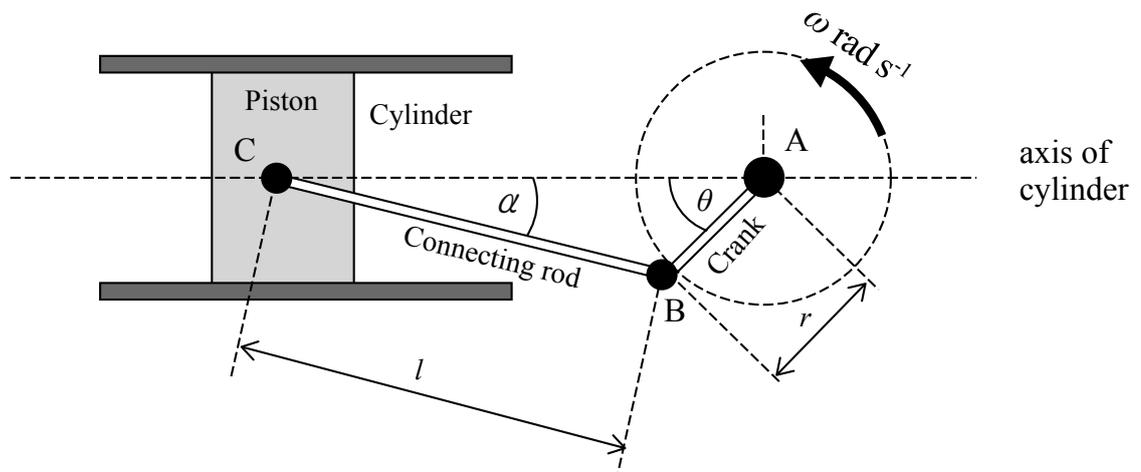


Fig. 7

The crank has radius  $r$  mm and rotates anticlockwise at a speed of  $\omega$  radians per second about the crankshaft which is indicated by point A. The connecting rod has length  $l$  mm and connects the crank to the piston at the pivot points B and C. As the crank rotates the piston moves backwards and forwards along the cylinder. When the crank makes an angle of  $\theta$  with the axis of the cylinder the connecting rod makes an angle of  $\alpha$  with the axis of the cylinder.

- (i) Show that  $\alpha = \sin^{-1}\left(\frac{r}{l} \sin \theta\right)$ .

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Fig. 8 shows velocity vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{s}$  associated with the mechanism shown in Fig. 7.

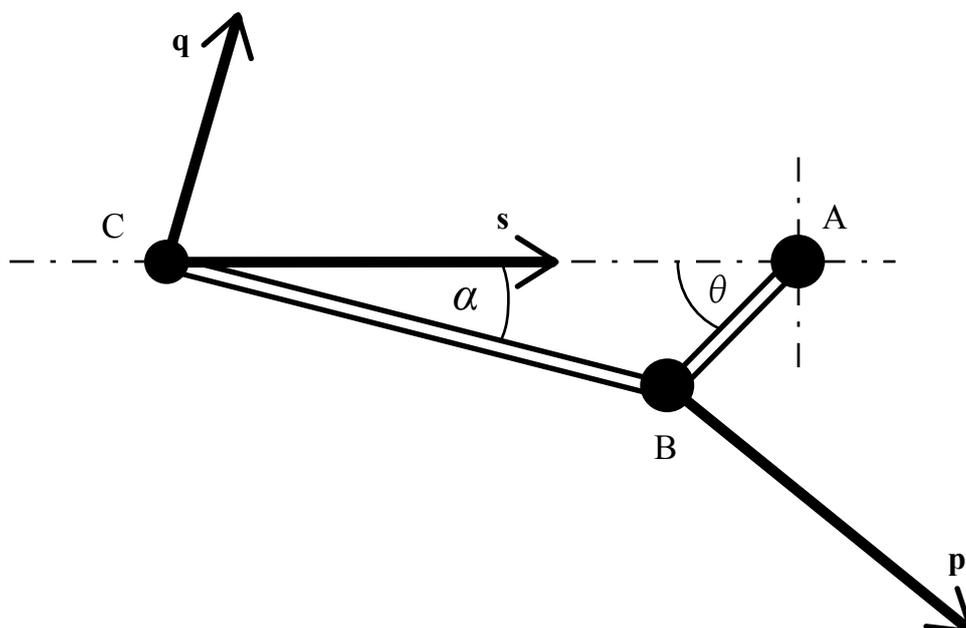


Fig. 8



(iv) Given that  $r = 50$ ,  $l = 175$  and  $\omega = 200$  calculate the speed of the piston along the cylinder when  $\theta = 35^\circ$ .

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..... [3]

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**Question 6 begins on page 16**



(iii) Calculate the time taken after the element is switched on for the temperature of the water to reach  $70^{\circ}\text{C}$ .

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- 7 (i) A parachutist jumps from a plane and falls under the influence of gravity until her speed reaches  $40 \text{ m s}^{-1}$  vertically downward. Assuming that aerodynamic drag can be neglected while she is falling, calculate the time taken to reach this speed after jumping from the plane. You may assume that her initial vertical speed is zero.

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- (ii) When her speed is  $40 \text{ m s}^{-1}$  she opens her parachute and continues downward for 100 seconds before landing on the ground. From the time when the parachute is first opened to the time when she lands on the ground her vertical speed,  $v \text{ m s}^{-1}$ , is modelled by

$$v = 30 \left( \frac{5t + 12}{3t^2 + 17t + 10} \right) + 4,$$

where  $t$  is the time in seconds after the parachute was opened.

Show that  $\left( \frac{5t + 12}{3t^2 + 17t + 10} \right) = \frac{A}{t + 5} + \frac{B}{3t + 2},$

where  $A$  and  $B$  are constants to be determined.

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