



Cambridge Technicals

Engineering

Unit **23**: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering

05822 - 05825

Mark Scheme for January 2018

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

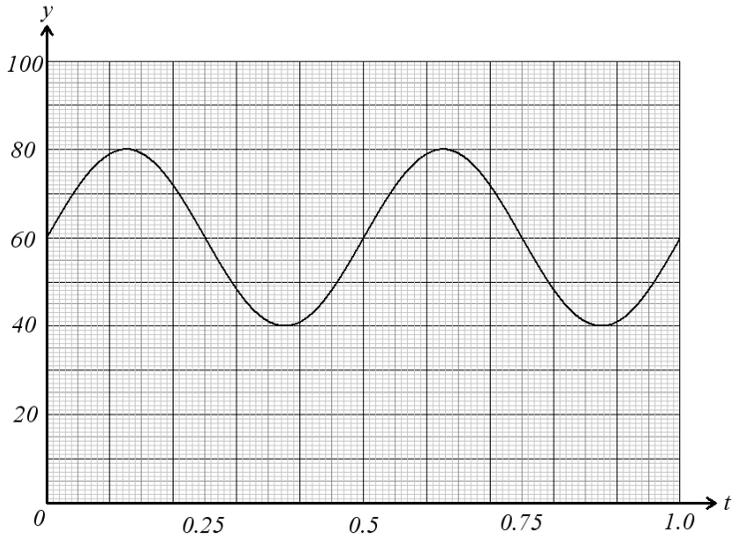
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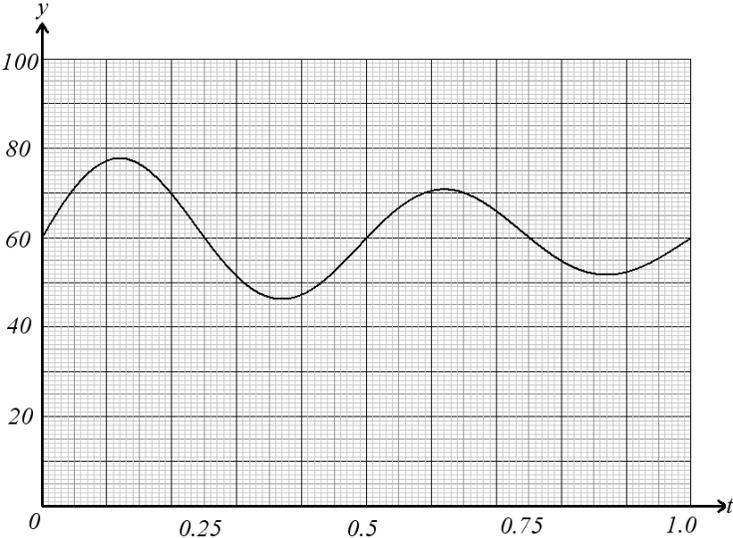
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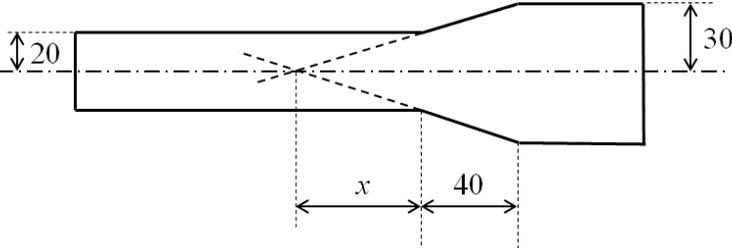
Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	Meaning
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

Question		Answer	Marks	Guidance
1	i	Volume flow rate = flow rate \times Area $v_1 A_1 = v_2 A_2$ $v_1 \pi \left(\frac{d_1}{2}\right)^2 = v_2 \pi \left(\frac{d_2}{2}\right)^2$ $v_1 d_1^2 = v_2 d_2^2$ $v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2$	M1 A1	
1	ii	$v = 15 \left(\frac{1}{0.5}\right)^2 = 60 \text{ m s}^{-1}$ [1]	B1	
1	iii	$h = \frac{v^2}{2g} = 183.67(346..) \approx 184$	M1 A1	M mark soi by correct value of h . Accept answer rounded to 184 PE at standardisation consider allow ecf from (ii) for v ?
1	iv	Volume flow rate $V = vA = 60\pi 0.25^2 = 11.78(097..) \text{ m}^3 \text{ s}^{-1}$ Mass flow rate $Q = V \times 1000 = 11780.(972..) \text{ kg s}^{-1}$	M1 A1 A1	M mark soi by correct values of V and Q . Alternatively $vA = 15\pi 0.5^2 = 11.78(097..)$ PE at standardisation consider allow ecf from (ii) for v and also calculation of Q using their V ? 2 marks targeted at Pass.
1	v	$P = \eta Qgh$ $\eta = 20\,000\,000 / (11780 \times 9.8 \times 183.67) = 0.943(140..) \approx 94\%$	M1 A1	M mark soi by correct value of η . Accept use of 184 for h and answer rounded to 94% PE at standardisation to allow ecf from previous answers.

Question			Answer	Marks	Guidance										
2	i		Simple harmonic motion.	B1	Also accept (undamped) sinusoidal oscillation. Do not accept terms such as 'sine wave', 'periodic function', 'periodic oscillation' etc.										
2	ii	<i>A</i>	4π (2 cycles) : $f = 2$ cycles per second	B1											
2	ii	<i>B</i>	$T = 1/f = 0.5$ s	B1											
2	iii			B1 Correct labels and values B1 Two cycles of a sine wave B1 Centred around 60 oscillating between 80 and 40											
2	iv	<i>A</i>	<table border="1" data-bbox="347 1189 974 1268"> <tbody> <tr> <td><i>t</i></td> <td>0.125</td> <td>0.375</td> <td>0.625</td> <td>0.875</td> </tr> <tr> <td><i>y</i></td> <td>77.65</td> <td>46.25</td> <td>70.71</td> <td>51.66</td> </tr> </tbody> </table>	<i>t</i>	0.125	0.375	0.625	0.875	<i>y</i>	77.65	46.25	70.71	51.66	B3	One mark for each correct value of <i>y</i> rounded up or down..
<i>t</i>	0.125	0.375	0.625	0.875											
<i>y</i>	77.65	46.25	70.71	51.66											

Question			Answer	Marks	Guidance
2	iv	B		B2	<p>Clear damped shape indicated [1] No mark if straight lines are used.</p> <p>Curve/lines goes approximately through points in table [1]</p> <p>NOTE: The actual maximum and minimum values of y occur a short time before the values in the table but do not penalise candidates if this is not indicated in the graph.</p> <p>PE at standardisation to allow ecf for answers for y on part A?</p>
2	iv	C	<p>Accept answers between 0.31 ± 0.03 AND between 0.43 ± 0.03</p>	B2	<p>Accept ECF from part (iv)(b) provided values correspond to an incorrect graph shown e.g cosine wave, non-damped wave, graph with straight lines etc.</p>

Question	Answer	Marks	Guidance
3 i	<p>Surface area of section on the left = $\pi DL = \pi 40 \times 150 = 6000\pi$ Surface area of section on the right = $\pi DL = \pi 60 \times 50 = 3000\pi$</p> <p>Extend the sloping side of central section x mm back to the centre of the tube.</p>  <p>Use similar triangles $\frac{20}{x} = \frac{30 - 20}{40} \Rightarrow x = 80$</p> <p>Sloping side of extended cone = $\sqrt{30^2 + (40 + 80)^2} = \sqrt{15300} = 123.69(3168..)$</p> <p>Sloping side of cone extension = $\sqrt{20^2 + 80^2} = \sqrt{6800} = 82.46(2112..) [1]$</p> <p>Surface area of central section = $\pi(30\sqrt{15300} - 20\sqrt{6800}) = 2061.55(2813...) \pi [1]$</p> <p>Total surface area = $(6000 + 3000 + 2062)\pi \approx 34751 \text{ mm}^2 [1]$</p>	<p>M1 M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>No diagram is required for full marks.</p> <p>All M marks soi by correct answers.</p>

Question	Answer	Marks	Guidance
3 ii	<p>The calculation assumes that the inside surface area is the same as the outside surface area (or similar explanation).</p> <p>This will include additional material that does not exist and so the result will be too large.</p> <p>An accurate method would be to subtract the inside volume of the tube from the total volume of the tube.</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>Accept explanation with reference to an annotated diagram if given.</p>
4 i	$F(x) = 0.01x^2 - 0.4x + 10 - (-0.001x^2 - 0.06x)$ $h = 0.011x^2 - 0.34x + 10$	<p>M1</p> <p>A1</p>	<p>Allow M mark soi by correct answer</p>
4 ii	$\frac{dh}{dx} = 0.022x - 0.34 = 0$ $x = \frac{0.34}{0.022} = 15.45(45\dots)$ $\frac{d^2h}{dx^2} = 0.022 > 0 \text{ so a minimum}$ $h = 0.011 \times 15.4545^2 - 0.32 \times 15.4545 + 10 = 7.37(27, \dots)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p>	<p>PE at standardisation to allow ecf from (i)</p> <p>4 synoptic marks – Unit 1, LO 5.1</p>
4 iii	$\int_0^{30} 0.011x^2 - 0.34x + 10 dx$ $\left[\frac{0.011}{3} x^3 - \frac{0.34}{2} x^2 + 10x \right]_0^{30}$ $= 99 - 153 + 300 = 246$	<p>M2</p> <p>A2</p> <p>A1</p>	<p>Also accept</p> $\int_0^{30} 0.01x^2 - 0.4x + 10 dx - \int_0^{10} -0.001x^2 - 0.06x dx$ <p>PE at standardisation to allow ecf from (i)</p>

Question		Answer	Marks	Guidance
5	i	$\frac{r}{\sin \alpha} = \frac{l}{\sin \theta}$ $\sin \alpha = \frac{r}{l} \sin \theta$ $\alpha = \sin^{-1} \left(\frac{r}{l} \sin \theta \right)$	<p>M1</p> <p>M1</p> <p>A1</p>	1 synoptic mark Unit 1, LO 4.4
5	ii		B3	<p>3 synoptic marks Unit 2, LO 2.1</p> <p>1 mark for each correct angle</p>
5	iii	$\frac{ \mathbf{p} }{\sin(90-\alpha)} = \frac{ \mathbf{s} }{\sin(\alpha + \theta)}$ $ \mathbf{s} = \frac{ \mathbf{p} \sin(\alpha + \theta)}{\cos \alpha}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Accept ecf from part (ii)</p> <p>Accept any equivalent answer</p> <p>e.g. $\mathbf{s} = \frac{ \mathbf{p} \sin(C)}{\cos A}$</p>

Question	Answer	Marks	Guidance
5 iv	$ \mathbf{p} = \omega r = 200 \times 50 \text{ mm s}^{-1} = 10000 \text{ mm s}^{-1}$ $\alpha = \sin^{-1}\left(\frac{50}{175} \sin 35\right) = 9.43^\circ$ <p>Using $\mathbf{s} = \frac{ \mathbf{p} \sin(\alpha + \theta)}{\cos \alpha}$</p> $ \mathbf{s} = \frac{10000 \sin(9.43 + 35)}{\cos 9.43} \approx \frac{7000}{0.9865} \approx 7095 \approx 7.1 \text{ m s}^{-1}$	<p>B1</p> <p>B1</p> <p>B1</p>	
6 i	$\frac{d^2 T}{dt^2} = -0.7e^{-\frac{t}{10}}$ $\frac{dT}{dt} = -0.7 \times (-10)e^{-\frac{t}{10}} + C = 7e^{-\frac{t}{10}} + C \quad [2]$ $7e^{-\frac{0}{10}} + C = 7 \Rightarrow C = 0 \quad [1]$ $\frac{dT}{dt} = 7e^{-\frac{t}{10}}$ $T = 7 \times (-10)e^{-\frac{t}{10}} + K = -70e^{-\frac{t}{10}} + K \quad [1]$ $-70e^{-\frac{0}{10}} + K = 10 \Rightarrow K = 80 \quad [1]$ $T = -70e^{-\frac{t}{10}} + 80 = 10 + 70(1 - e^{-\frac{t}{10}}) \quad [1]$	<p>M2</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Do not allow solutions that start with $T = 10 + 70(1 - e^{-\frac{t}{10}})$</p> <p>and differentiate to get $\frac{d^2 T}{dt^2} = -0.7e^{-\frac{t}{10}}$</p>

Question		Answer	Marks	Guidance
7	i	using $v = u + at$ from formula booklet with $u = 0$, $v = 40$ and $a = g = 9.8$ $t = \frac{40}{9.8} = 4.08(16\dots)$ s [1]	M1 A1	
7	ii	$\left(\frac{5t+12}{3t^2+17t+10} \right) = \frac{5t+12}{(t+5)(3t+2)} = \frac{A}{t+5} + \frac{B}{3t+2}$ $5t+12 = A(3t+2) + B(t+5)$ $t = -\frac{2}{3} \Rightarrow -5 \times \frac{2}{3} + 12 = B \left(-\frac{2}{3} + 5 \right)$ $-10 + 36 = B(-2 + 15)$ $13B = 26 \quad B = 2$ $t = -5 \Rightarrow -5 \times 5 + 12 = A(-15 + 2)$ $-13A = -13 \quad A = 1$	M1 A1 A1	Accept use of the cover-up rule seen from. $\frac{5t+12}{(t+5)(3t+2)} = \frac{A}{t+5} + \frac{B}{3t+2}$ for M1

Question		Answer	Marks	Guidance
7	iii	$\int_0^{100} 30 \left(\frac{1}{t+5} + \frac{2}{3t+2} \right) + 4 dt$ $\left[30 \left(\ln(t+5) + \frac{2}{3} \ln(3t+2) \right) + 4t \right]_0^{100} \quad [3]$ $30(4.654 + 3.807) + 400 - 30(1.609 + 0.462) = 591.7 \quad [3]$	<p>M3</p> <p>A2</p>	
7	iv	<p>Distance travelled in free fall using</p> $s = ut + \frac{at^2}{2} = 0 + \frac{9.8 \times 4.0816^2}{2} = 81.63(27\dots)$ <p>OR using $v^2 = u^2 + 2as$</p> $s = 0 + \frac{v^2}{2g} = \frac{40^2}{2 \times 9.8} = 81.63(27\dots)$ <p>height = 81.6 + 591.7 = 673.3 m</p>	<p>M1</p> <p>A1</p>	<p>Condone arithmetic errors provided a correct formula with correct values is seen.</p> <p>Accept ecf from part iii</p>

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