OCR
Oxford Cambridge and RSA

Cambridge Technicals

Engineering

Unit **23:** Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations

Annotation	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	**Meaning**
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

Subject-specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.
Mark in using a red pen.
Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.
Transfer these marks to the box on the front page.
Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM
A method mark which is dependent on a previous method mark.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

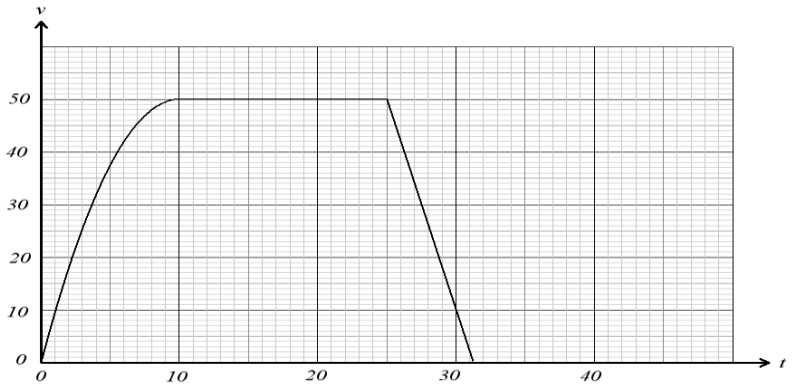
Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		Plate 1 Volume of material A = $(0.2^2 - 0.1^2) \times 0.05 = 0.0015$ oe Volume of material B = $0.1^2 \times 0.05 = 0.0005$ oe Plate 2 Volume of material A = $(0.3^2 - 0.2^2) \times 0.05 = 0.0025$ oe Volume of material B = $0.2^2 \times 0.05 = 0.002$ oe Plate 1 mass $0.0015a + 0.0005b = 10$ oe Plate 2 mass $0.0025a + 0.002b = 25$ oe $(0.0015a + 0.0005b = 10) \times 1000/5 \rightarrow 3a + b = 20000$ $(0.0025a + 0.002b = 25) \times 1000/5 \rightarrow 5a + 4b = 50000$ AG	**M1** **M1** **M1** **M1** **DM1** **A1** **[6]**	Expect 1 500 000 Expect 500 000 Expect 2 500 000 Expect 2 000 000 Must have consistent RHS If zero: SCB1 for area of material A on either plate (30 000 or 50 000)
1	(ii)		$\begin{bmatrix}3 & 1\\5 & 4\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}20000\\50000\end{bmatrix}$	**B1** **[1]**	3 matrices required. Multiplication LHS
1	(iii)		$= \frac{1}{7}\begin{bmatrix}4 & -1\\-5 & 3\end{bmatrix}\begin{bmatrix}20000\\50000\end{bmatrix}$ $\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}3 & 1\\5 & 4\end{bmatrix}^{-1}\begin{bmatrix}20000\\50000\end{bmatrix}$ $= \frac{1}{7}\begin{bmatrix}30000\\50000\end{bmatrix} = \begin{bmatrix}4286\\7143\end{bmatrix}$ $a = 4386$ $b = 7143$	**B1** **B1** **M1** **A1** **A1** **[5]**	B1 for $\frac{1}{7}$ or 7 soi B1 for $\begin{bmatrix}4 & -1\\-5 & 3\end{bmatrix}$ soi Soi with *their* determinant 4285.71 7142.86 No marks for non-matrix methods

Question			Answer	Marks	Guidance
2	(i)		$2+\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}=4$ $2+\dfrac{1}{\dfrac{R_2+R_1}{R_1R_2}}=4 \quad 2+\dfrac{R_1R_2}{R_1+R_2}=4$ AG	**M1** **A1** **[2]**	Answer must show clear understanding of resistances in series and parallel. Reciprocal step must be convincing as AG. (May use power -1)
2	(ii)		$\dfrac{1}{\dfrac{1}{2}+\dfrac{1}{R_1}}+R_2=4.5$ $\dfrac{1}{\dfrac{R_1+2}{2R_1}}+R_2=4.5 \quad \dfrac{2R_1}{R_1+2}+R_2=4.5$ AG	**M1** **A1** **[2]**	Answer must show clear understanding of resistances in series and parallel. Reciprocal step must be convincing as AG. (May use power -1)
2	(iii)		$[\dfrac{R_1R_2}{R_1+R_2}=2]$ $\quad R_1R_2=2(R_1+R_2)$ oe $R_2(R_1-2)=2R_1$ $R_2=\dfrac{2R_1}{R_1-2}$ oe	**M1** **M1** **A1** **[3]**	Clear all fractions by appropriate method Collect R_2 terms and factorise
2	(iv)		$[\dfrac{2R_1}{R_1+2}+\dfrac{2R_1}{R_1-2}=4.5]$ $2R_1(R_1-2)+2R_1(R_1+2)=4.5(R_1-2)(R_1+2)$ $2R_1^{\ 2}-4R_1+2R^2{}_1+4R_1=4.5(R_1^{\ 2}-4)$ $0.5R_1^{\ 2}=18$ $R_1=\sqrt{36}=6$ cao	**M1** **M1** **M1** **A1** **[4]**	All method marks are available following substitution of *their* R_2 Substitute and clear all fractions by appropriate method Expand all terms on both sides as required Expect $4R_1^{\ 2}=4.5R_1^{\ 2}-18$ Not dependent on earlier marks. Attempt to solve their equation in R_1

Question			Answer	Marks	Guidance
2	(v)		$R_2 = \frac{2R_1}{R_1 - 2} = \frac{12}{4} = 3$	**B1 FT [1]**	*their* R_1 and *their* expression for R_2

Question			Answer		Marks	Guidance
3	(i)		$tan\theta_1 = \frac{h}{AC}$ and $tan\theta_2 = \frac{h}{BC}$	$tan\theta_1 = \frac{h}{AC}$ and $tan\theta_2 = \frac{h}{BC}$	**B1**	oe soi
			Horizontal approach:	Vertical approach:		
			$AC = \frac{h}{tan\theta_1}$ and $BC = \frac{h}{tan\theta_2}$	$h = ACtan\theta_1$ and $h = BCtan\theta_2$	**B1**	oe soi
			$[AC = BC + 15]$	$(BC + 15)tan\theta_1 = BCtan\theta_2$	**M1**	Establish relationship using their expressions
			$\frac{h}{tan\theta_1} = \frac{h}{tan\theta_2} + 15$	$\left(\frac{h}{tan\theta_2} + 15\right)tan\theta_1 = \frac{h}{tan\theta_2}tan\theta_2$	**A1**	oe correct equation in h only
			$h(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}) = 15$	$15tan\theta_1 = h\left(1 - \frac{tan\theta_1}{tan\theta_2}\right)$	**M1**	Collect terms and factorise
			$h = \frac{15\tan\theta_1\tan\theta_2}{\tan\theta_2 - \tan\theta_1}$ AG	$h = \frac{15\tan\theta_1\tan\theta_2}{\tan\theta_2 - \tan\theta_1}$ AG	**A1**	
					[6]	
3	(ii)		$[h =] 87$		**B1**	87.1158
			$[AC =] 139$		**B1**	139.413
			$\theta_3 = tan^{-1}\frac{87.1}{69.7}$		**M1**	soi *their* h and *their* $\frac{1}{2}AC$
			$51.3°$		**A1**	
					[4]	

Question			Answer	Marks	Guidance
4	(i)		$10 - t = 0$ $v = 10 \times 10 - 0.5 \times 10^2$ 50 [m/s] AG	M1 DM1 A1 [3]	Derivative seen, equated to zero Soi using *their* t Candidates do not need to prove this is a maximum value
4	(ii)		6.25 [s]	B1 [1]	
4	(iii)			B1 B1 B1 B1 [4]	Labels and appropriate scale on both axes Curve from 0 to 50 between 0 and *their* t Constant line at 50 between *their* t and *their* $t + 15$ Straight line from 50 to 0 FT time obtained in ii
4	(iv)		Distance d1= distance in first period $\int_0^{10} 10t - 0.5t^2 \mathrm{d}t = \left[5t^2 - \frac{0.5t^3}{3}\right]_0^{10}$ $= 500 - \frac{500}{3} = (333.33)$ d2= distance in second period $50 \times 15 = 750$ d3= distance in third period $\frac{1}{2}(50) \times 6.25 = 156.25\,\text{m}$ total distance = 333.33 +750 + 156.25 ≈1240 m	M1 A1 B1 B1 B1 [5]	attempt at integration SC B3 1156.25 from 3 straight line sections

Question			Answer	Marks	Guidance
5	(i)		$x_1 = 10$	B1 [1]	
5	(ii)		$2 \times 5cos45$ or $2 \times 5sin45$ $x_2 = x_1 + 7.1$ Expect 17.1	M1 A1 FT [2]	soi by 7.07 or $5\sqrt{2}$ Accept alternatives eg $\sqrt{5^2 + 5^2}$ or correct use of sine rule FT from *their (i)*
5	(iii)	(*A*)	$h = a(X + x_2)^2 + b(X + x_2) + c$ $h = aX^2 + (2ax_2 + b)X + (ax_2^2 + bx_2 + c)$	B1 B1 [2]	Substitution seen or clearly implied by expansion of brackets Expansion and collection of terms to required format must be seen
5	(iii)	(*B*)	[When $X = 0$, $h = 10$ therefore] $E = 10$. $\frac{\mathbf{d}h}{\mathbf{d}X} = 2aX + D$ When $X = 0$, $\frac{\mathrm{d}h}{\mathrm{d}X} = -1$ therefore $D = -1$	B1 B1 [2]	Seen Obtained using differentiation
5	(iii)	(*C*)	Gradient $\frac{\mathrm{d}h}{\mathrm{d}X} = 2aX - 1$ $2aX - 1 = 0 \quad \rightarrow \quad a = \frac{1}{2X}$ Height $aX^2 - X + 10 = 0$ Substituting $a = \frac{1}{2X}$ and solving for X $20 + 17.071 = 37.071$	M1 M1 M1 M1 A1 FT [5]	Correct form for derivative FT their D FT their D and E must equate to zero Expect $X = 20$ Only FT their answer to ii

Question			Answer	Marks	Guidance
6	(i)		$h = 12 \Rightarrow t = \frac{3\pi}{2}$ $h = 0 \Rightarrow t = -\frac{\pi}{2}$ $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{\pi}{6}$: $\mathrm{d}h = \frac{6}{\pi}\mathrm{d}t$ $\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} 400(\sin t + 1)\frac{6}{\pi}\mathrm{d}t = \frac{2400}{\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin t + 1\,\mathrm{d}t$ AG	**B1** **M1** **DM1** **A1** **[4]**	Both limits seen with unsimplified integrand in t only and dt present No method marks available without clear attempt at $\frac{dt}{dh}$
6	(ii)		$\frac{2400}{\pi}\left[-\cos t + t\right]_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}$ $\frac{2400}{\pi}\left\{(-\cos(\frac{3\pi}{2}) + \frac{3\pi}{2}) - (-\cos(-\frac{\pi}{2}) - \frac{\pi}{2})\right\}$ [2] $\frac{2400}{\pi}\left\{(0 + \frac{3\pi}{2}) - (0 - \frac{\pi}{2})\right\} = \frac{2400}{\pi} \times 2\pi = 4800$ [2] 4800 [Wh m^{-2}]	**B1 B1** **M1** **DM1** **A1** **[5]**	Each part must be seen in terms of t Coefficient not required until final A1. Limits not required at this point Substitution of both limits into *their* integrated function involving t Evaluation from 4 terms Oe

Question			Answer	Marks	Guidance
7	(i)		$1 \times 0.5 \times h = 0.005$ $h = 0.005/0.5 = 0.01$ m (10 mm)	M1 A1 [2]	Consistent units eg $1000 \times 500 \times h = 5000$ Alternative approaches acceptable, award M1 at point where height can be evaluated
7	(ii)		60s	B1 [1]	
7	(iii)	(*A*)	1 l = 2 [mm] Drain rate = $0.01h$ l s^{-1} = $0.01\times2\ h$ mm s^{-1} = $0.02\ h$ mm s^{-1} fall in height $\frac{dh}{dt} = -0.02h$ AG	M1 A1 [2]	Accept alternative reasoning e.g. $\frac{\mathbf{d}v}{\mathbf{d}t} = -0.01h$ (stated in the question) but $v = lwh = 1\times0.5\times h = 0.5h$ so $\frac{\mathbf{d}v}{\mathbf{d}t} = \frac{\mathbf{d}0.5h}{\mathbf{d}t} = 0.5\frac{\mathbf{d}h}{\mathbf{d}t} = -0.01h \Rightarrow \frac{\mathbf{d}h}{\mathbf{d}t} = -0.02h$
7	(iii)	(*B*)	$\int \frac{dh}{0.02h} = -\int 1\,dt$; $\int \frac{50}{h}dh = -\int 1\,dt$ $50\ln h = -t + c$ h = 600 when t = 0 –> c = $50\ln 600$ $t = 50\ln 600 - 50\ln h$; $t = 50(\ln 600 - \ln h)$ when h =200 $t = 50(\ln 600 - \ln 200)$ $t = 50\ln 3$ (= 54.9306...)	M1 M1 M1 M1 M1 M1 A1 [7]	Accept alternative solutions e.g. $\int \frac{dh}{0.02h} = -\int 1\,dt$ $\frac{\ln 0.02h}{0.02} = -t + k$ $\ln 0.02h = -0.02t + k_1$ $h = k_2 e^{-0.02t}$ h = 600 when t = 0 –> k_2 =600 $200 = 600e^{-0.02t}$ $-0.02t = \ln\frac{1}{3}$ t = 54.93 s

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