OCR
Oxford Cambridge and RSA

Level 3 Cambridge Technical in Engineering
05823/05824/05825/05873

Unit 23: Applied mathematics for engineering

Monday 4 June 2018 – Afternoon

Duration: 2 hours
C305/1806

You must have:
- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name		Last Name	
Centre Number		Candidate Number	
Date of Birth	D D M M Y Y Y Y		

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- If additional answer space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working.
- Marks may be given for a correct method even if the answer is incorrect. An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **20** pages.

FOR EXAMINER USE ONLY	
Question No	**Mark**
1	/12
2	/12
3	/10
4	/13
5	/12
6	/9
7	/12
Total	**/80**

Answer **all** questions.

1 Two metal plates are each constructed of two materials, A and B. One plate is 200 mm square and has an inner section which is 100 mm square as shown in Fig. 1. The mass of this plate is 10 kg. The other plate is 300 mm square and has an inner section which is 200 mm square as shown in Fig. 2. The mass of this plate is 25 kg. The outer sections of both plates are made of material A while the inner sections are made of material B. Both plates have a uniform thickness of 50 mm.

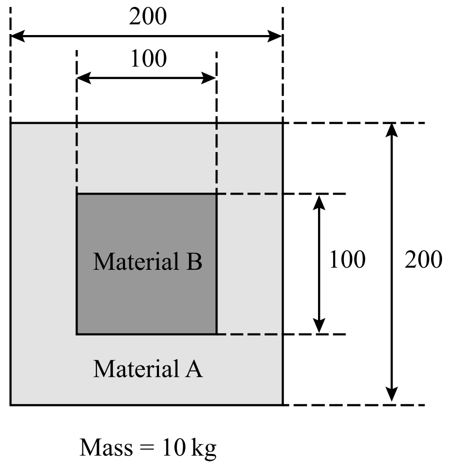

Fig. 1

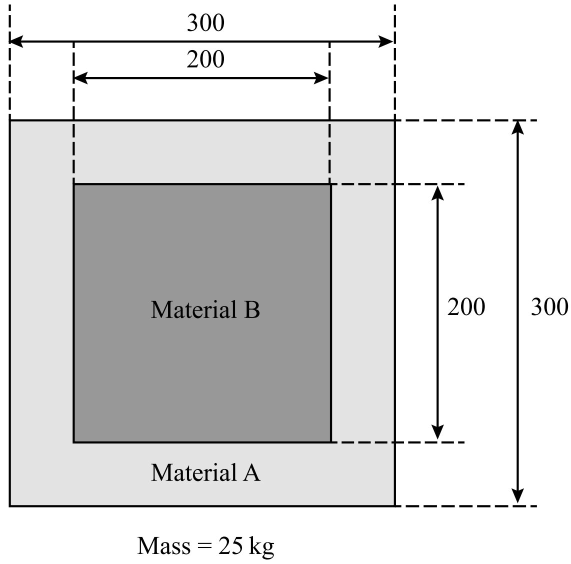

Fig. 2

The density of material A is a kg m^{-3} and the density of material B is b kg m^{-3}.

(i) Show that a and b satisfy the following linear simultaneous equations.

$$3a + b = 20\,000$$

$$5a + 4b = 50\,000$$

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.... **[6]**

(ii) Express these equations in matrix notation.

[1]

(iii) Use matrix methods to find *a* and *b*.

[5]

2 **(i)** An electrical circuit consists of a 2 Ω resistor connected in series with two other resistors with resistances R_1 Ω and R_2 Ω which are connected in parallel. The total resistance across this circuit is 4 Ω as shown in Fig. 3.

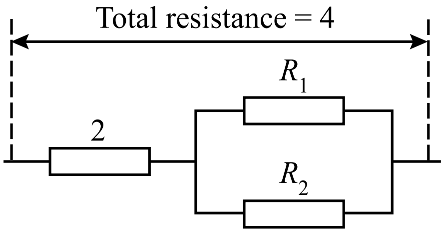

Fig. 3

Using the formulae for resistors in series and resistors in parallel given in the formula booklet provided, show that

$$2+\frac{R_1R_2}{R_1+R_2}=4. \qquad \text{(equation 1)}$$

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(ii) Another circuit consists of the same resistors connected in a different arrangement. In this circuit the 2 Ω resistor is connected in parallel with the R_1 Ω resistor; these are connected in series with the R_2 Ω resistor. The total resistance across this circuit is 4.5 Ω as shown in Fig. 4.

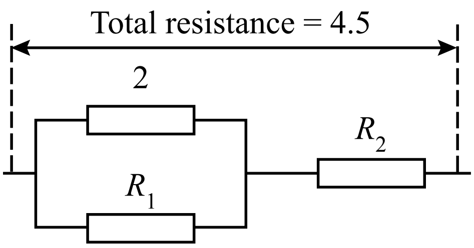

Fig. 4

Show that

$$\frac{2R_1}{R_1+2}+R_2=4.5. \qquad \text{(equation 2)}$$

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(iii) Rearrange equation 1 to make R_2 the subject.

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(iv) Substitute your expression for R_2 from part (iii) into equation 2 to find an equation in R_1 and hence find the value of R_1.

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(v) Find the value of R_2.

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3 A building with height h m is built on horizontal ground as shown in Fig. 5. Two points, A and B, are on the ground on one side of the building and are in line with point C at the base of the building. Points A and B are 15 m apart with point B being the nearer to the building. D is a point at the top of the building vertically above C. The angle of elevation of D from A is θ_1; the angle of elevation of D from B is θ_2.

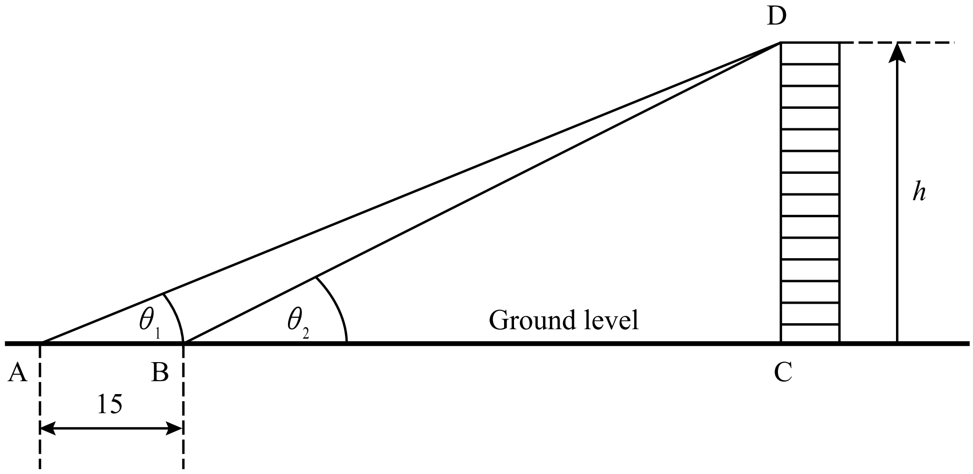

Fig. 5

(i) Show that the height of the building is given by

$$h = \frac{15\tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1}.$$

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(ii) At a certain time on a sunny day, the shadow of the building extends to a point E, which is half way between A and C as shown in Fig. 6.

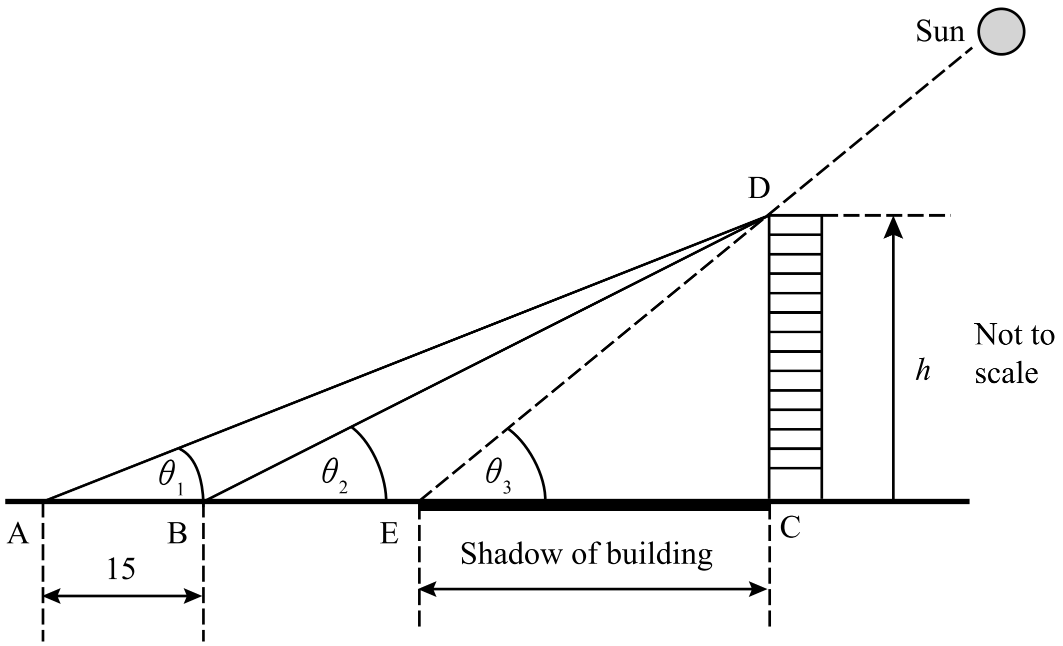

Fig. 6

Given that $\theta_1 = 32°$ and $\theta_2 = 35°$ calculate the angle of elevation, θ_3, of D from E.

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4 The performance of a new car is being evaluated on a test track. The car starts from rest and accelerates to its maximum speed. During this time the speed of the car, $v\,\mathrm{m\,s^{-1}}$, is modelled by the equation

$$v = 10t - 0.5t^2\,, \quad \text{where } t \text{ is time in seconds.}$$

Having reached its maximum speed the car continues without accelerating or decelerating for 15 s. At the end of this period the brakes are applied and the car decelerates uniformly at a rate of $8\,\mathrm{m\,s^{-2}}$ until it comes to rest.

(i) Show that the maximum speed reached by the car is $50\,\mathrm{m\,s^{-1}}$.

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(ii) Calculate the time during which the car is decelerating.

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(iii) Sketch a graph of speed against time on the grid below. Label the axes.

[4]

(iv) Calculate the total distance travelled by the car.

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5 Fig. 7 shows the side view of a roller coaster track consisting of three sections. The track and its supporting frame are aligned within Cartesian axes (x, h) with origin O and with the x-axis horizontal at ground level. All lengths are expressed in metres.

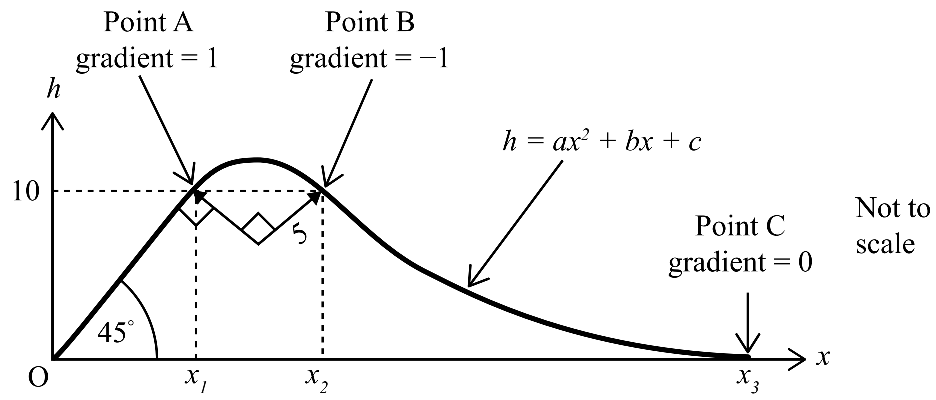

Fig. 7

The first section of the track, OA, has a constant gradient and makes an angle of 45° with the x-axis; the coordinates of A are (x_1, 10). The second section of the track, AB, is part of a semi-circle of radius 5; the coordinates of B are (x_2, 10). The third section of the track, BC, is part of the curve with equation

$$h = ax^2 + bx + c, \text{ where } a, b \text{ and } c \text{ are constants.}$$

The coordinates of C are (x_3, 0) and the gradient of the track at C is 0. In order to give a smooth ride the gradients of the track where the sections meet are continuous. The gradient at A is 1 and the gradient at B is −1.

(i) State the value of x_1.

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(ii) Calculate the value of x_2.

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(iii) ***(A)*** Use the substitution $x = X + x_2$ to show that the equation for the third section of the track can be written in the form $h = aX^2 + DX + E$, where D and E are constants.

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(B) Use the facts that, at point B, $X = 0$, $h = 10$ and the gradient of the track is -1, to find the values of D and E.

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(C) Use the facts that, at point C, $h = 0$ and the gradient of the track is zero, to calculate the value of x_3.

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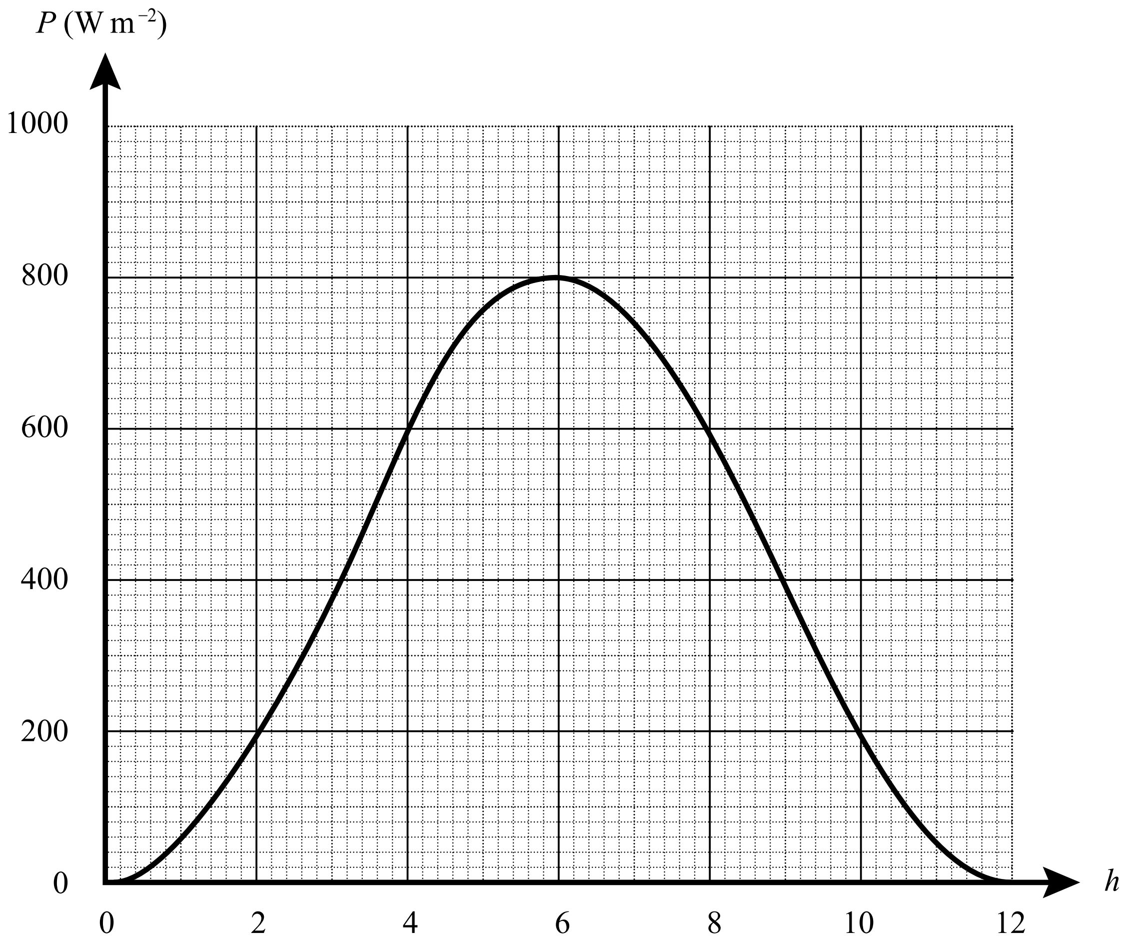
6 Solar irradiance is the power reaching the surface of the Earth from the Sun and can be measured in terms of Watts per square metre ($\mathrm{W\,m^{-2}}$). The power varies depending upon the month of the year, the time of the day and the atmospheric conditions, as well as the location on the Earth at which measurements are taken. On a clear summer's day at a particular location in southern England the solar irradiance ($P\,\mathrm{W\,m^{-2}}$) was measured on a horizontal surface for a 12-hour period between 6.00 am and 6.00 pm. The variation of solar irradiance during this period can be modelled by the equation

$$P = 400\left(\sin\left(\pi\left(\frac{h-3}{6}\right)\right) + 1\right), \quad \text{where } h \text{ is the number of hours after 6.00 am.}$$

A graph of P against h is shown below.



The total energy, $E\,\mathrm{W\,h\,m^{-2}}$ for this 12-hour period is given by

$$E = \int_0^{12} 400\left(\sin\left(\pi\left(\frac{h-3}{6}\right)\right) + 1\right)\mathrm{d}h.$$

(i) By substituting $t = \pi\left(\frac{h-3}{6}\right)$ show that

$$\int_0^{12} 400\,(\sin(\pi(\frac{h-3}{6})) + 1)\,dh = \frac{2400}{\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin t + 1)\,dt.$$

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(ii) Calculate the total energy, E Wh m^{-2}, for the 12-hour period by evaluating

$$\frac{2400}{\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin t + 1)\,dt.$$

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7 In this question you will need to use the fact that 1000 litres = 1 m^3.

A rectangular water tank has a horizontal base that is 1000 mm long and 500 mm wide.

(i) Calculate the height of water above the base of the tank when the tank contains 5 litres.

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(ii) Water is fed into the tank at a rate of 5 litres per second. Starting with an empty tank, calculate the time taken for the water to reach a height of 600 mm above the base.

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(iii) At the base of the tank there is a drain valve. When this valve is opened it allows water to drain at a rate that is directly proportional to the height, h, of water above the base of the tank, measured in mm. In this case the drain rate is $0.01h$ litres per second.

(A) Show that, when the water supply is turned off and the drain valve is open,

$$\frac{dh}{dt} = -0.02h.$$

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(B) The height of the water above the base of the tank is 600 mm and the water supply is turned off. The drain valve is opened. Calculate the time taken for the height of water above the base of the tank to fall from 600 mm to 200 mm.

[7]

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional answer space is required, you should use the following lined pages. The question number(s) must be clearly shown – for example 1(ii) or 4(i).