OCR
Oxford Cambridge and RSA

Thursday 17 January 2019 – Afternoon

LEVEL 3 CAMBRIDGE TECHNICAL IN ENGINEERING

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Duration: 2 hours

C305/1901

C305/1901

You must have:
- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name		Last Name	

Centre Number						Candidate Number				

Date of Birth	D	D	M	M	Y	Y	Y	Y

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **16** pages.

FOR EXAMINER USE ONLY	
Question No	**Mark**
1	/11
2	/12
3	/11
4	/13
5	/10
6	/12
7	/11
Total	**/80**

Answer **all** the questions

1 For this question you may wish to refer to sections 6.3 and 6.4 of the formula book provided.

The term 'lift force' when applied to an aircraft refers to the upward force exerted on its wings as they pass through the air. The lift force, L N, of a particular aircraft while it is travelling along a horizontal runway passing through still air is given by the equation

$L = 100v^2$, where v m s^{-1} is the speed of the aircraft.

When the aircraft accelerates along the runway to take off it must exceed a speed V m s^{-1} at which the magnitude of the lift force is equal to the total weight of the aircraft, including its load. The total mass of the aircraft, including its load, is 50 000 kg.

(i) Calculate V.

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(ii) While the aircraft is accelerating along the runway the engines provide a constant forward thrust of 150 kN. Neglecting any retarding forces, and assuming that the aircraft has constant acceleration starting from rest,

(A) Calculate the time taken for the aircraft to reach speed V m s^{-1},

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(B) Calculate the distance travelled in this time.

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(C) Comment on your answers to parts (*A*) and (*B*) with reference to any simplifying assumptions made.

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2 In this question take acceleration due to gravity to be 10 m s^{-2}, so $g = 10$.

Fig. 1 shows an arrangement in which a mass M of 50 kg is supported in equilibrium by three cables. The cables pass over fixed frictionless pulleys and are attached to masses of m_1 kg, m_2 kg and 20 kg.

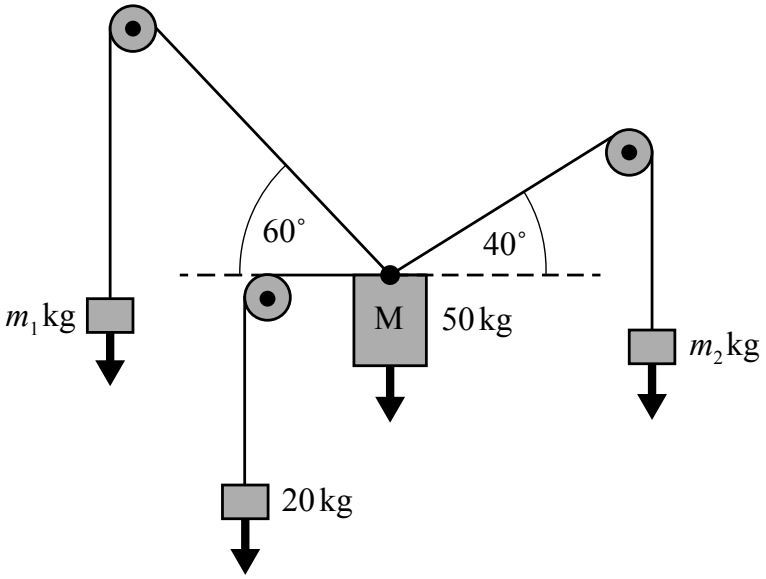

Fig.1

(i) Sketch a diagram of a quadrilateral showing the forces acting on M as a closed chain of four vectors. Indicate on the diagram the magnitude of each vector in terms of m_1, m_2 and known values.

[2]

(ii) Write each of the four vectors in part **(i)** in the form $a\mathbf{i} + b\mathbf{j}$, where **i** and **j** are perpendicular unit vectors and a and b are expressions involving m_1, m_2 and given values.

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(iii) Calculate the values of m_1 and m_2 so that mass M remains in static equilibrium.

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(iv) The vector product (also called the cross product) of two vectors **a** and **b** is a vector perpendicular to the plane containing **a** and **b** with magnitude

$$|\mathbf{a}||\mathbf{b}| \sin\theta,$$

where $|\mathbf{a}|$ and $|\mathbf{b}|$ are the magnitudes of vectors **a** and **b** respectively and θ is the angle between them.

Calculate the magnitude of the vector product of the two vectors in part **(ii)** that are associated with m_1 and m_2.

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3 The passive impedances, $Z\,\Omega$, of a resistor, an inductor and a capacitor when subjected to an input voltage of the form $v = V\sin(\omega t)$ are given below.

Resistor:	$Z_R = R$	where $R\,\Omega$ is resistance
Inductor:	$Z_L = \omega L\,\mathrm{j}$	where L H is inductance
Capacitor:	$Z_C = \dfrac{1}{\omega C\,\mathrm{j}}$	where C F is capacitance
(Note	$\mathrm{j} = \sqrt{-1}$)	

When the components are connected in series the total passive impedance, Z_T, is given by

$$Z_T = Z_\mathrm{R} + Z_\mathrm{L} + Z_\mathrm{C}.$$

Fig. 2 shows a circuit containing a resistor, an inductor and a capacitor connected in series subjected to an input voltage $v = V\sin(\omega t)$.

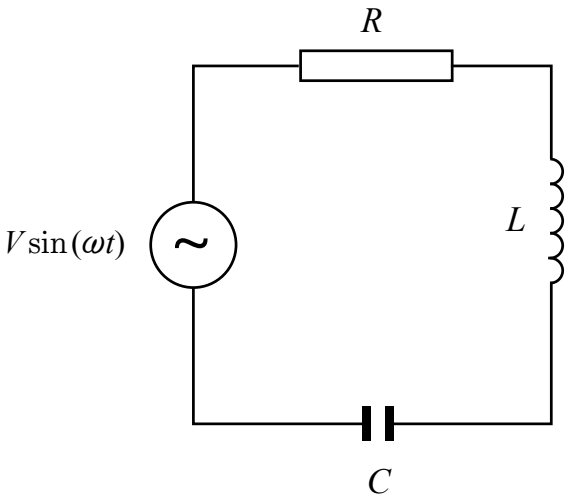

Fig. 2

(i) The input voltage, v, has a frequency of 50 Hz. Calculate the value of ω.

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(ii) You are also given following values.

$R = 20$

$L = 30 \times 10^{-3}$

$C = 40 \times 10^{-6}$

(A) Express Z_T in the form $a + b\text{j}$ where a and b are real values.

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(B) Represent Z_T on an Argand diagram.

[2]

(C) Express Z_T in the form $r(\cos\theta + \text{j}\sin\theta)$, where r is a real value and θ is an angle in degrees.

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(D) Express Z_T in exponential form, $r\text{e}^{\theta\text{j}}$.

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4 **(i)** Fig. 3 shows a circle of radius r with an angle of θ subtended by a chord of length c.

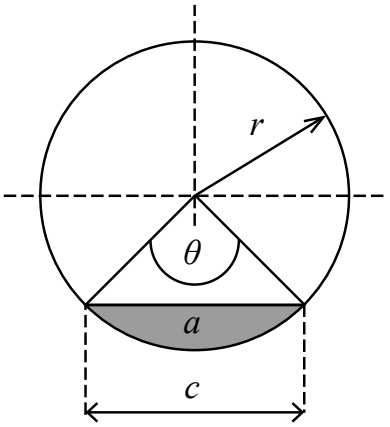

Fig. 3

(A) Show that $\theta = 2\sin^{-1}\frac{c}{2r}$.

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(B) Show that the shaded area, a, between the chord and the circumference of the circle is given by

$$a = \frac{r^2}{2}(\theta - \sin\theta).$$

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(ii)

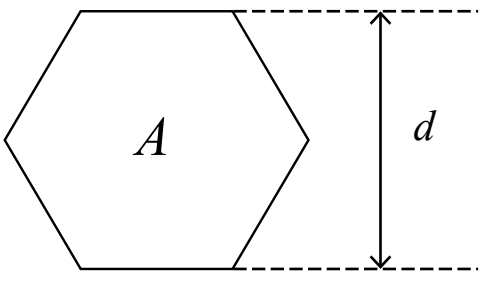

Fig. 4

Show that the area, A, of the regular hexagon shown in Fig. 4 is given by

$A = \frac{\sqrt{3}d^2}{2}$, where d is the distance between two parallel sides.

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(iii) Fig. 5 shows a 20 mm steel spanner with a thickness of 5 mm. The head of the spanner is circular with a diameter of 40 mm and contains a centrally located hexagonal hole. The handle attached to the head is 60 mm long and 20 mm wide.

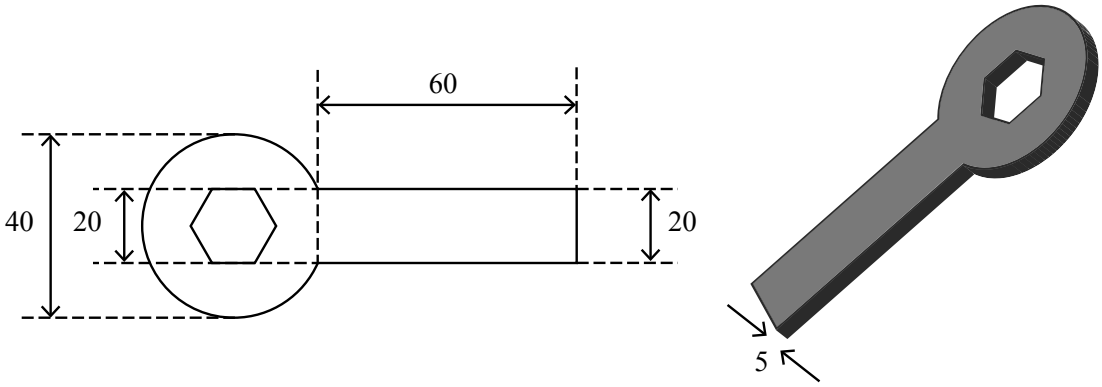

Fig. 5

Calculate the volume of steel in the spanner, giving your answer to the nearest cubic millimetre.

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5 The acceleration of a car, a m s^{-2}, is modelled by the equation $a = Ae^{-ct}$ where A and c are constants and t is time measured in seconds. The car is travelling at a speed of v_o m s^{-1} when $t = 0$. After t seconds the speed of the car has increased to v m s^{-1} and the car has travelled a distance of s m.

(i) Write the equation $a = Ae^{-ct}$ as a differential equation involving v.

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(ii) Solve the differential equation to find an expression for v in terms of t, A, c and v_o.

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(iii) Find an expression for the distance s in terms of t, A, c and v_o.

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(iv) You are given that $A = 8$, $c = 0.3$ and $v_o = 10$.
Calculate the time taken for the car to reach a speed of 34 m s^{-1}.

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6 The alternating current, I A, drawn by a particular electronic circuit at time t s is given by

$$I = e^{-10t}\sin(50t), \text{ for } t \geq 0.$$

(i) Sketch a graph of I against t for the interval $0 \leq t < 0.2$.

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(ii) Using calculus, calculate the time at which current I first becomes a maximum.

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(iii) Calculate the minimum value of I.

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7 The relationship between pressure, p MPa, and volume, V litres, within a combustion chamber of an engine may be modelled by the Otto cycle. A graph of an idealised Otto cycle for a particular four-stroke petrol engine is shown in Fig. 6.

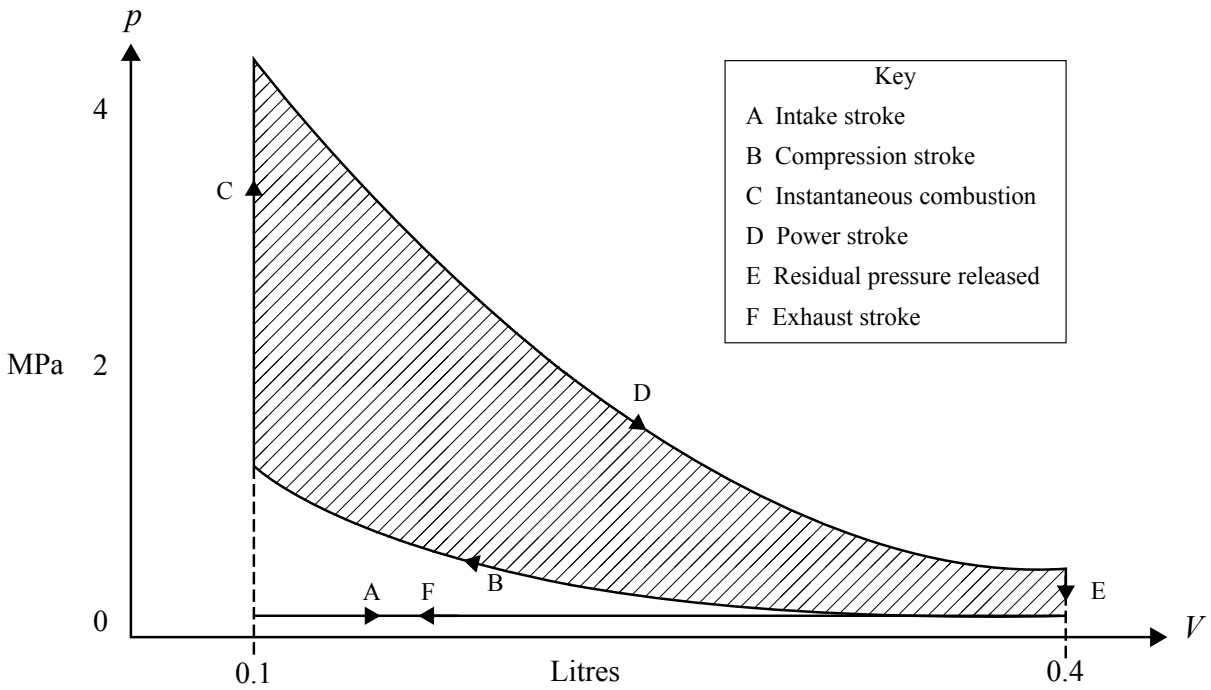

Fig. 6

The curves representing the power stroke and the compression stroke have been modelled by the following equations.

$$\left.\begin{array}{l}\text{Power stroke (curve D) } p = 39V^2 - 32V + 7 \\ \text{Compression stoke (curve B) } p = 3\text{e}^{-12V}\end{array}\right\} \quad 0.1 \leq V \leq 0.4$$

(i) Calculate the value of V on the power stroke when the pressure is 2 MPa.

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(ii) The shaded area shown in Fig. 6 between the two curves representing the power stroke and the compression stroke, bounded by the minimum and maximum volumes, is the work done, W kilojoules (kJ), by the piston in two revolutions of the engine.

Calculate W.

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END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional answer space is required, you should use the following lined pages. The question number(s) must be clearly shown – for example 5(i) or 7(ii).

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