OCR
RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

FSMQ **6993**

Mark Schemes for the Units

June 2009

6993/MS/R/09

Oxford Cambridge and RSA Examinations

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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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Foundations of Advanced Mathematics FSMQ (6993)

MARKSCHEME ON THE UNIT

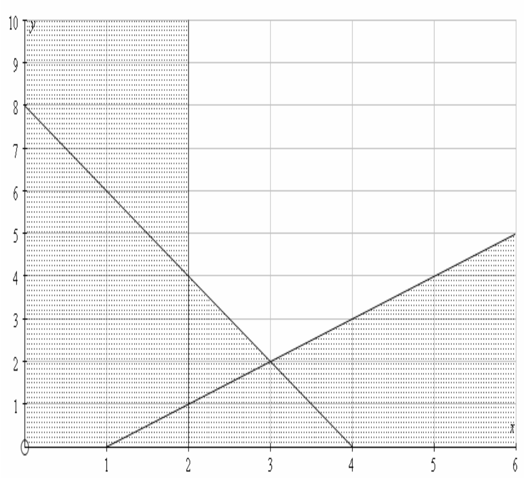
Additional Mathematics – 6993

Section A

1		Pythagoras for third value: $c=\sqrt{5}$ $\Rightarrow \tan\theta = -\frac{\sqrt{5}}{2}$	M1 A1 A1 **3**	Using any means to find $\sqrt{5}$ Includes negative sign.
		Alt: $\sin^2\theta = 1-\cos^2\theta = 1-\frac{4}{9}=\frac{5}{9}$ $\Rightarrow \sin\theta = \frac{1}{3}\sqrt{5}$ $\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}/3}{2/3} = -\frac{\sqrt{5}}{2}$	M1 A1 A1	Use of Pythagoras Sin θ Includes negative sign
		SC: Allow B1 for $\tan\theta = -1.12$		
2		$\frac{dy}{dx} = 3x^2+5$ $\Rightarrow$ grad tangent $=8$ $\Rightarrow$ grad normal $=-\frac{1}{8}$ $\Rightarrow y+1 = -\frac{1}{8}(x-1)$ $\Rightarrow 8y+8=-x+1 \Rightarrow 8y+x+7=0$	M1 A1 F1 M1 A1 **5**	Attempt at differentiation with at least one term with correct power Dep on use of their normal gradient and correct point Any acceptable form. Acceptable means three terms only
3	(i)	$2x+5y=2+25$ $\Rightarrow 2x+5y=27$	M1 A1 **2**	Substitute new point to change c If put in form $y=mx+c$ then $m=-0.4$
		SC: B2 from scale drawing only if absolutely correct		
	(ii)	When $x=3,\ 6+5y=27$ $\Rightarrow 5y=21 \Rightarrow y=\frac{21}{5}$ $\Rightarrow p=\frac{21}{5}=4.2$	M1 F1 **2**	Substituting $x=3$ into either their equation from (i) **or** the given equation in (i) Answer must specifically give p
		NB $p=0.2$ comes from using original line. Give M1 A1 for this.		

4	(i)	$AB = \sqrt{(5-1)^2 + (3-1)^2}$ $= \sqrt{4^2 + 2^2}$ $= \sqrt{20} = 2\sqrt{5}$	M1 A1 2	 isw ie ignore any approx value for root.
		NB M1 A0 for 4.47 with no sight of $\sqrt{20}$		
	(ii)	$\left(\frac{1+5}{2}, \frac{1+3}{2}\right) = (3,2)$	B1 1	
	(iii)	$(x \pm a)^2 + (y \pm b)^2$ with (a,b) from **(ii)** $(x-a)^2 + (y-b)^2$ $= 5$	M1 F1 A1 3	Use of equation Their midpoint cao for 5 isw ie ignore any incorrect algebra following a correct equation
5	(i)	$v^2 = u^2 + 2as \Rightarrow 0 = 4 - 2 \times 0.25s$ $\Rightarrow s = 8$ Distance travelled = 8 m	M1 A1 A1 3	Use of right formula(e) Substitution Answer
		If t is found first then M1 for any correct equations that lead to finding s Careful also of $4 = 0 + \frac{1}{2}s$, this could be 3 if quoted formula is right. Also of $0 = 4 + \frac{1}{2}s \Rightarrow s = -8$ Both of these M1 for formula only		
	(ii)	$s = ut + \frac{1}{2}at^2 = s = 3 \times 4 - \frac{1}{2} \times 0.25 \times 16$ $= 12 - 2 = 10$ Length of ramp = 10 m	M1 A1 A1 3	
		NB Anything that uses $v = 0$ is M0		
6		$\frac{dy}{dx} = 1 - 4x + 3x^2$ $\Rightarrow (y =) x - 2x^2 + x^3 (+c)$ Through (2, 6) $\Rightarrow 6 = 2 - 8 + 8 + c \Rightarrow c = 4$ $\Rightarrow y = x - 2x^2 + x^3 + 4$	M1 A1 M1 A1 4	For integrating - increase in power of one in at least two terms Attempt to find c Must be an equation

7	**(i)**	$AC^2 = 8^2 + 3^2 - 2.8.3.\cos 60$ $= 73 - 24 = 49$ $\Rightarrow AC = 7$ $\Rightarrow$ Total distance = 18 km	M1 A1 A1 F1 **4**	Use of formula AC Total distance
	(ii)	$\frac{\sin \text{BCA}}{8} = \frac{\sin 60}{9}$ $\Rightarrow \sin \text{BCA} = \frac{8}{9} \times \sin 60 (= 0.7698)$ $\Rightarrow \text{BCA} = 50.3^0$	M1 A1 A1 **3**	
		Alternative Scheme: Use of cosine formula twice $\Rightarrow$ BC = 9.74… $\Rightarrow$ BCA = 50.3^0	 M1 A1 A1	
8		$2x + 11 = x^2 - x + 5$ $\Rightarrow x^2 - 3x - 6 \ (= 0)$ $\Rightarrow x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2}$ $= 4.37$ or -1.37	M1 A1 M1 A1 A1 **5**	Substitute Quadratic Solve Correct substitution Both answers Ignore values for y
		Alternative Scheme 1 (relates to last 3 marks) Completion of square: $(x - 1.5)^2 = k$ $x - 1.5 = \pm\sqrt{8.25}$ $\Rightarrow x = 4.37$ or -1.37	 M1 A1 A1	 Must contain ± Must be 2 dp
		Alternative Scheme 2: Only 2 marks from last 3 Solving their quadratic by T&I Both roots	 M1 A1	
		Alternative Scheme 3. Only 4 marks Roots with no working: B2 each	 B2,2	
		Alternative Scheme 4. Only 4 marks Finding a root from the original equations = one of them Finding the second root = the other	 M1 A1 M1 A1	
		Alternative scheme 5. Eliminate x. Gives y^2 - $28y$ + 163 = 0 Gives y = 19.74 and 8.26 leading to x values	M1 A1 M1 A1 A1	Eliminate x Quadratic Solve Both y values Both x values
		NB Attempt to solve by graph - M0		

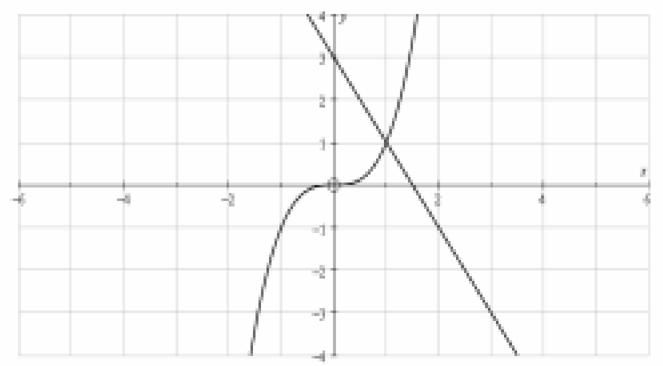
9	(i)	$a = 4 - 0.2t$ $\Rightarrow v = 4t - 0.1t^2$ $\Rightarrow v_5 = 20 - 2.5 = 17.5$ Velocity is 17.5 m s^{-1}	M1 A1 A1 **3**	Integrate (increase of power of one in at least one term) Ignore c
	(ii)	At $t = 20$, $a = 0$ ie Maximum velocity	B1 **1**	
	(iii)	$v = 4t - 0.1t^2$ $\Rightarrow s = \int_0^{20} 4t - 0.1t^2 \, dt = \left[2t^2 - 0.1\frac{t^3}{3}\right]_0^{20}$ $= 2 \times 400 - 0.1 \times \frac{8000}{3} = 533.3... = 533$ Distance travelled = 533 m	M1 A1 A1 **3**	Integrate their v from (i) (Increase in power of one term) Ignore c Allow exact answer or 3sf
10	(i)		B2,1 B2,1 **4**	Lines, −1 each error Shading, −1 each error Correct side of line. ft if gradient is the same sign.
	(ii)	$y = 2$	E1 **1**	ft their graph

Section B

11	**(i)**	$-x^2+8x-9=x^2-6x+11$ $\Rightarrow 2x^2-14x+20=0$ $\Rightarrow x^2-7x+10=0$ $\Rightarrow (x-5)(x-2)=0$ $\Rightarrow x=2,\ 5$ Substitute: $x=2\Rightarrow y=4-12+11=3$ $x=5\Rightarrow y=25-30+11=6$	M1 A1 M1 A1 A1 **5**	Equate Quadratic Solve: Factorisation needs 2 numbers to multiply to their constant Or one pair, e.g. (2,3) or (5,6)
		Alternative scheme: Completion of square: $(x-3.5)^2=k$ $x-3.5=\pm\sqrt{2.25}$ $\Rightarrow x=5$ or 2 $\Rightarrow y=6$ or 3	M1 A1 A1	
	(ii)	$A=\int_2^5 (y_1-y_2)\,dx=\int_2^5\left(-2x^2+14x-20\right)dx$ $=\left[-\frac{2x^3}{3}+7x^2-20x\right]_2^5$ $=\left(-\frac{2\times125}{3}+7\times25-100\right)-\left(-\frac{16}{3}+28-40\right)$ $=\left(-\frac{250}{3}+75\right)-\left(-\frac{16}{3}-12\right)=-\frac{234}{3}+87=87-78=9$	M1 A1 M1 A2 M1 A1 **7**	Int between curves ± Correct expression Integrate their function (not if they divide by 2) All terms, −1 for each error Sub into integral Answer
		Alternative scheme: $A=\int_2^5\left(-x^2+8x-9\right)dx-\int_2^5\left(x^2-6x+11\right)dx$ $=\left[-\frac{x^3}{3}+4x^2-9x\right]_2^5-\left[\frac{x^3}{3}-3x^2+11x\right]_2^5$ $=\left(\left(-\frac{125}{3}+100-45\right)-\left(-\frac{8}{3}+16-18\right)\right)$ $-\left(\left(\frac{125}{3}-75+55\right)-\left(\frac{8}{3}-12+22\right)\right)$ $=\left(13\frac{1}{3}-\left(-4\frac{2}{3}\right)\right)-\left(21\frac{2}{3}-12\frac{2}{3}\right)=18-9$ $=9$	M1 M1 A1 A1 M1 A1 A1	Subtracting 2 integrals Integrate either All terms of y_1 All terms of y_2 Substitute into either integral For 18 or 9 Final answer
		SC $A=\int(y_1+y_2)\,dx$ M1 integrate and M1 sub only		

12	**(i)**	$\frac{100}{BE} = \sin 30$ $\Rightarrow BE = \frac{100}{\sin 30} = 200$ m	M1 A1 A1 **3**	Fraction right way up Correct expression for BE Or B3 if the special triangle is noticed.
		Alternative scheme: $\frac{100}{BC} = \tan 30 \Rightarrow BC = \frac{100}{\tan 30} = 173.2$ $BE = \sqrt{100^2 + 173.2^2} = 200$	M1 A1 A1	Ratio and Pythagoras Allow not exact
	(ii)	AE by Pythagoras: $AE = \sqrt{500^2 + 200^2} = 100\sqrt{29} = 538.5...$ $\sin A = \frac{100}{538.5}$ $\Rightarrow A = 10.7^0$	M1 A1 M1 A1 **4**	soi
		Alternative Scheme: $BC = \sqrt{30000} \approx 173.2 \Rightarrow AC = \sqrt{280000} \approx 529.2$ $\Rightarrow A = \tan^{-1}\frac{100}{\sqrt{280000}} = 10.7^0$ NB $A = 10.9^0$ comes from $\sin^{-1}\frac{100}{\sqrt{280000}}$	M1 A1 M1 A1	
	(iii)	Area $= \frac{1}{2} \times 500 \times$ their BE $= 50\,000$ Area $= \frac{1}{2} \times BG \times$ their AE $\Rightarrow BG = \frac{2 \times \text{their area}}{\text{their AE}} = 185.7...... \approx 186$ m	M1 A1 M1 A1 A1 **5**	
		Alternative Scheme: Find angle A or E Then $\frac{BG}{500} = \sin A \Rightarrow BG = 186$ m ie maximum 3 marks. The answer is found, but the question says "Hence" and this is "otherwise". NB If area is attempted but not used then give M1 A1. If area is found after BG is found then do not mark it.	M1 A1 A1	

		In all parts of this question allow answers to 3sf or 4 dp		
13	**(a)**	The selection is random. *Allow anything that implies equal chance of selection*	B1 **1**	
	(b)(i)	P(all are female) = 0.6^6 (= 0.046656) = 0.0467	M1 A1 **2**	Sight of 0.6^6 Must be 3 sf
	(ii)	P(3 of each) = Bin coeff × $0.6^3 \times 0.4^3$ = 20 × $0.6^3 \times 0.4^3$ = 0.2765 or 0.276	M1 A1 A1 A1 **4**	One term with binomial coeff 20 (may be implied) Powers (may be implied)
	(iii)	P(more females than males) = 6, 0 or 5,1 or 4,2 = $0.6^6 + 6 \times 0.6^5 \times 0.4 + 15 \times 0.6^4 \times 0.4^2$ = 0.04666 + 0.1866 + 0.3110 = 0.5443 Allow 0.544, 0.545, 0.5444	M1 B1 B1 B1 A1 **5**	Add 3 terms Binomial coefficients correct in at least two terms Powers correct in at least two terms At least 2 terms correct.
		Alternative scheme: P(more females than males) = 1 – P(more males than females *or* equal numbers) = 1 – (0.4^6+6×0.4^5×0.6+15×0.4^4×0.6^2+20×0.4^3 ×0.6^3 = 1 – (0.0041 + 0.0369 + 0.1382 + 0.2765) = 0.5443	M1 B1 B1 B1 A1	Take 4 terms from 1 Binomial coeffs Powers At least 2 terms correct
		The terms are: 0.0467, 0.1866, 0.3110, 0.2765, 0.1382, 0.0369, 0.0041		
		If P(more males than females), treat as MR and –2 If p = 0.4 and q = 0.6 then MR –2 (but also 0 for (b)(i) where answer is given!)		

14	**(a)(i)**		B1 B1 **2**	Line with +ve intercepts and –ve gradient Curve Condone +ve gradient for cubic at origin. Must pass through the origin
	(ii)	Can only intersect in one point.	B1 **1**	Allow if obviously true, even if one or both are wrong
		NB Do not allow if the curve implies that there could be more than one root but the line has not been drawn long enough - eg if curve is quadratic		
	(b)(i)	$\frac{dy}{dx} = 3x^2 + 3$ Greater than 0 for all x or attempt to solve their $\frac{dy}{dx} = 0$ so no solution to $3x^2 + 3 = 0$	B1 M1 A1 **3**	Correct two terms $= 0$ No solution
	(ii)	Because the curve is always increasing can only cross the x axis in one point which is the root	B1 **1**	There must be some reference to **(b)(i)**
	(c)(i)	By trial f(2) = 0 Condone $(x - 2)$ is a factor	B1 **1**	
	(ii)	$\Rightarrow (x - 2)(x^2 + 2x + 5) = 0$	M1 A1 **2**	In long division at least x^2 must be seen
	(iii)	Discriminant "$b^2 - 4ac$" = -16 < 0 So no roots. This means that $x = 2$ is the only root.	B1 **1**	Depends on **(ii)** being correct
		NB "Quad does not factorise" is not good enough		
	(d)	The equation will only have one root (for all r and s.)	B1 **1**	Ignore extra comments even if wrong

Grade Thresholds

Additional Mathematics (6993)
June 2009 Assessment Series

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	73	63	53	44	35	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	27.7	39.7	48.7	56.9	66.0	100	9859

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