

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation	Meaning
✓	Tick
✗	Cross
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0	Method mark awarded 0
M1	Method mark awarded 1
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded 1
B0	Independent mark zero
B1	Independent mark 1
SC	Special case
^K	Omission mark
MR	Misread

Subject-specific Marking Instructions

- 1 M (method) marks are not lost for purely numerical errors.
A (accuracy) marks depend on preceding M (method) marks. Therefore M0 A1 cannot be awarded.
B (independent) marks are independent of M (method) marks and are awarded for a correct final answer or a correct intermediate stage.
- 2 Subject to 2, two situations may be indicated on the mark scheme conditioning the award of A marks or independent marks:
 - i. Correct answer correctly obtained (no symbol)
 - ii. Follows correctly from a previous answer whether correct or not (" on mark scheme and on the annotations tool).
- 3 Always mark the greatest number of significant figures seen, even if this is then rounded or truncated in the answer.
- 4 Where there is clear evidence of a misread, a penalty of 1 mark is generally appropriate. This may be achieved by awarding M marks but not an A mark, or awarding one mark less than the maximum.
- 5 Where a follow through () mark is indicated on the mark scheme for a particular part question, you must ensure that you refer back to the answer of the previous part question if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Abbreviations

The following abbreviations are commonly found in Mathematics mark schemes.

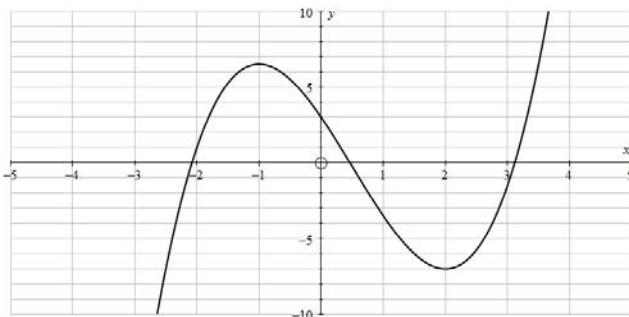
- Where you see **oe** in the mark scheme it means **or equivalent**;
- Where you see **cao** in the mark scheme it means **correct answer only**;
- Where you see **soi** in the mark scheme it means **seen or implied**;
- Where you see **www** in the mark scheme it means **without wrong working**;
- Where you see **rot** in the mark scheme it means **rounded or truncated**;
- Where you see **seen** in the mark scheme it means that you should award the mark if that number/expression is seen anywhere in the answer space, even if it is not in the method leading to the final answer;
- Where you see **figs 237**, for example, this means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point eg 237000, 2.37, 2.370, 0.00237 would be acceptable but 23070 or 2374 would not.

Section A

Question		Answer	Marks	Rationale
1	(i)	Gradient = -1.5	M1 A1 2	Attempt to rearrange to to $y = \dots$ Mark final answer (Ans = $-1.5x$ is M1A0)
		<p><i>Alternative method:</i></p> <p>Find two points on line and then the gradient by $\frac{\Delta y}{\Delta x}$ M1 (The two points must lie on the line) Ans A1</p>		
1	(ii)	<p>Gradient = $\frac{2}{3}$</p> <p>Use (3,1) and <i>their</i> normal gradient in a standard form for a line $\Rightarrow 3y = 2x - 3$ oe</p>	M1 M1 A1 3	soi There must be an attempt to find a normal gradient for this method mark to be earned 3 terms only
		<p><i>Alternative method:</i></p> <p>$2x - 3y = k$ B1 Sub (3,1) M1 Ans A1</p>		

Question		Answer	Marks	Rationale
2		$\begin{aligned} -7 < 3x + 1 < 12 \\ \Rightarrow -8 < 3x < 11 \\ \Rightarrow -\frac{8}{3} < x < \frac{11}{3} \\ \Rightarrow -2, -1, 0, 1, 2, 3 \end{aligned}$	M1 M1 A1 A1 4	Subtract either side by 1 (or $\frac{1}{3}$ if done the other way round) Divide throughout by 3 Or $-\frac{8}{3} < x$ and $x < \frac{11}{3}$ Mark final answer SC1 for $x < \frac{11}{3}$ or $x > -\frac{8}{3}$ but not both
		Alternative method: Can be done by trial: B4 all correct One missing or one extra B2 Mark final answer		
3		John's age, $4x$ soi Ages in 5 yrs $x + 5$, $4x + 5$ soi $\begin{aligned} 4x + 5 &= 3(x + 5) \\ 4x + 5 &= 3x + 15 \\ x &= 10 \quad \text{No ISW} \end{aligned}$	B1 B1 M1 A1 4	Condone use of their own letters SC2 Answer only with no equation formed or incorrect algebra
		Alternative method: Forming simultaneous equations: Correct equations implies B1, B1 Eg $j = 4p$ and $j + 5 = 3(p + 5)$ Give M1 only if there is an attempt to eliminate one variable.		

Question		Answer	Marks	Rationale
4		$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{5}{9} = \frac{4}{9}$ $\Rightarrow \cos\theta = \frac{2}{3}$ $\Rightarrow \tan\theta = \frac{1}{2}\sqrt{5} \text{ ISW}$	M1 A1 A1 3	Any use of calculators to approximate gets A0
		<i>Alternative method:</i> Find third side of triangle M1 = 2 A1 Ans A1		

Question		Answer	Marks	Rationale
5	(i)	$y = x^3 - \frac{3}{2}x^2 - 6x + 3$ $\Rightarrow \frac{dy}{dx} = 3x^2 - 3x - 6$ $\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$ $\Rightarrow x = 2, -1$ $\Rightarrow y = -7, 6.5$ $\Rightarrow (2, -7), (-1, 6.5)$	M1 A1 M1 A1 A1 5	Diffn – all powers reduced by 1 – allow one error (beware dividing by x) All terms correct Set = 0 and solve (dependent on first M mark) Both x Both pairs (Allow this mark if the coordinates are explicitly stated in 5(ii).)
5	(ii)		B1 B1 2	Correct shape (Cubic the right way up and two turning points, does not need three intercepts on x -axis) Through (0, 3), (2, -7) and (-1, 6.5). Dep on 1st B mark Allow y-intercept in range [2, 4] At $x = 2$ allow y in range [-6, -8] At $x = -1$ allow y in range [6, 8]

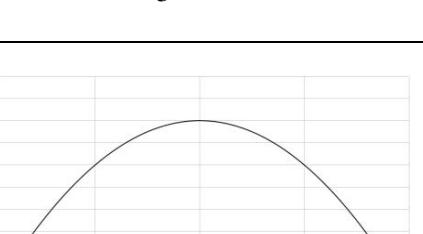
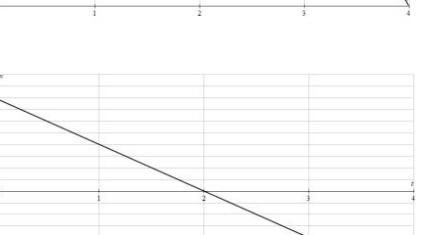
Question		Answer	Marks	Rationale
6	(i)	$\begin{aligned} P(2 \text{ from } 3) &= \binom{3}{2} p^2 q \\ &= 3 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) \\ &= \frac{5}{72} \text{ or } 0.0694\ldots \text{ oe ISW} \end{aligned}$	B1 B1 B1 3	1 term with correct fractions and powers Correct coefficient of 3 attached to their probability term soi At least 3 sf Correct answer only B3
6	(ii)	$\begin{aligned} P(\text{at least } 1) &= 1 - P(0) = 1 - q^3 \\ &= 1 - \left(\frac{5}{6}\right)^3 \\ &= \frac{91}{216} \text{ or } 0.421\ldots \end{aligned}$	M1 A1 A1 3	At least 3 sf
		<i>Alternative method:</i> $\begin{aligned} P(1) + P(2) + P(3) & \quad \text{M1 Add three terms} \\ &= 0.3472 + 0.0694 + 0.0046 \\ &= 0.421\ldots \quad \text{A1 Three terms correct soi} \\ & \quad \text{A1} \end{aligned}$		
7		Sin rule: $\frac{\sin 40}{4} = \frac{\sin B}{6}$ $\Rightarrow \sin B = \frac{6 \times \sin 40}{4} (= 0.964\ldots)$ $\Rightarrow B = 74.6^\circ \text{ or } 105.4^\circ$ and 105.4° or 74.6°	M1 A1 A1 B1 4	Sin rule or complete method via the perpendicular to find angle Correctly applied in this case soi One value FT $180 - \text{their } B$, unless $B = 90$

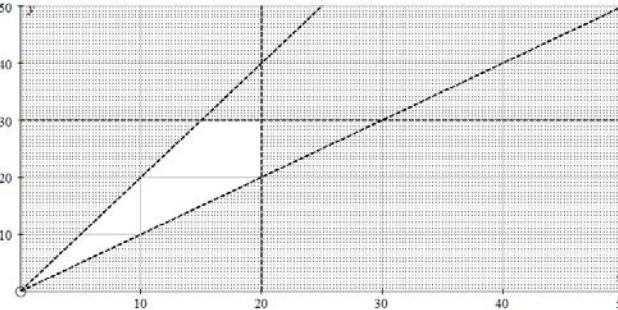
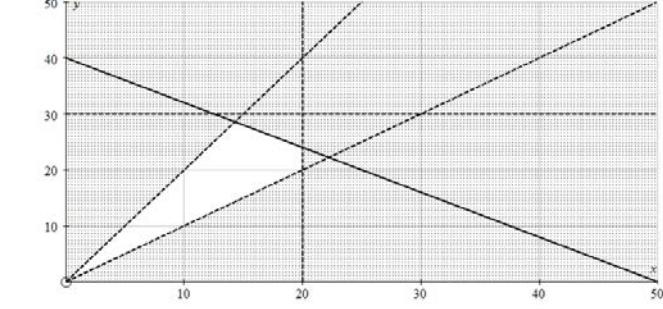
Question		Answer	Marks	Rationale
8		$\int_0^8 (8x - x^2) dx = \left[4x^2 - \frac{x^3}{3} \right]_0^8$ $= 4 \times 8^2 - \frac{8^3}{3} = 64 \times \frac{4}{3}$ $= \frac{256}{3} \text{ or } 85\frac{1}{3} \text{ or } 85.3 \text{ or better}$ <p>Total area = 160</p> $\Rightarrow \text{Proportion} = \frac{\frac{256}{3}}{160} = \frac{256}{480} = 53.3\%$	M1 A1 M1 A1 M1 A1 6	Increase in power of 1 in at least one term (beware multiplying by x) Both terms Substitute limits of 0 and their upper limit following integration (NB Limits the wrong way round M1 A0) Dep on previous M, ratio of their ans / 160 Dep on previous A1 (NOT 53) NB The answer is given.
		<p>Alternative method:</p> <p>53% of 160 M1 dep on all other M marks $= 84.8$</p> <p>The two answers related, eg both corrected to 2 sf = 85 A1</p>		Dep on previous A1
9	(i)	$x^2 + 8x + 19 = (x + 4)^2 + 19 - 16$ $= (x + 4)^2 + 3$	M1 A1 A1 3	Attempt to complete square or expand rhs to give quadratic expression For 4 www For 3 NB For completion of square at least $(x \pm 4)^2$ seen
9	(ii)	When $x = -$ their a Value is their b	B1 B1 2	FT FT
9	(iii)	$\frac{1}{\text{their } b}$	B1 1	FT

Question		Answer	Marks	Rationale
10	(i)	$\text{Angle CAB} = \tan^{-1} \frac{2400}{1000} = 67.4^\circ \text{ or bearing} = 067^\circ$ $\text{Distance} = \sqrt{1000^2 + 2400^2} = 2600$ (Rounds to 2600 to 4 sf)	M1 A1 M1 A1 4	Mark final answer (Don't allow 67) Can be awarded if seen in part (ii)
10	(ii)	$AD = \frac{1000}{\cos 55}$ or anything that rounds to 1743 $CD = 1000 \tan 55$ or anything that rounds to 1428 $DB = 2400 - CD = 972$ $AD + DB - AB =$ anything that rounds to 115(m)	B1 B1 B1 3	Don't accept premature approximation cao www Correct answer www B3
		<i>Alternative method:</i> Sine rule on right-hand triangle $\frac{x}{\sin 22.6} = \frac{y}{\sin 12.4} = \frac{2600}{\sin 145} \quad \mathbf{B1}$ (all values seen, angles rounding to 12.4 and 22.6) $x = 1742 - 1744$ $y = 971 - 973 \quad \mathbf{B1}$ both values Answer $\mathbf{B1}$		Accept a combination

Section B

Question		Answer	Marks	Rationale	
11	(a)	Radius 10	B1		
		Centre (2, 0)	B1		
11	(b)	(i)	<p>Substitute $y = 2x + 6$ into $(x-2)^2 + y^2 = 100$</p> $\Rightarrow (x-2)^2 + (2x+6)^2 = 100$ $\Rightarrow 5x^2 + 20x + 40 = 100$ $\Rightarrow x^2 + 4x - 12 = 0$ $\Rightarrow (x+6)(x-2) = 0$ $\Rightarrow A \text{ is } (-6, -6) \text{ and } B \text{ is } (2, 10)$	M1 A1 M1 A1 A1 5	Substitute 3 term quadratic Solve a 3 term quadratic Either both x or both y or one pair Both pairs
11	(b)	(ii)	$\text{Midpoint AB} = \left(\frac{-6+2}{2}, \frac{-6+10}{2} \right) \Rightarrow (-2, 2)$	B1 1	
11	(b)	(iii)	$\text{AB} = \sqrt{(-6-2)^2 + (-6-10)^2}$ $= \sqrt{8^2 + 16^2} = \sqrt{320} = 8\sqrt{5} \text{ or } 17.9$	M1 A1 2	
11	(c)		$\text{distance} = \sqrt{\text{radius}^2 - \text{half their (iii)}^2}$ $= \sqrt{100 - 80} = \sqrt{20} = 2\sqrt{5} \text{ or } 4.47$	M1 A1 2	
			<p><i>Alternative method:</i> Their centre to their midpoint $= \sqrt{(2-(-2))^2 + 2^2} = \sqrt{20}$</p>	M1A1	

Question		Answer	Marks	Rationale
12	(a)	$v = 8t - 2t^2$ $v = 0 \text{ when } 8t = 2t^2 \Rightarrow t = 4$ or when $t = 4$, $v = 8 \times 4 - 2 \times 4^2 = 32 - 32 = 0$	M1 A1 M1 A1 4	Diffn v Final answer Set = 0 or $t=4$, dep on first M1 SC3 for confirming = 0 when v divided by 2
12	(b)	(i)	M1 A1 M1 A1 4	Diffn <i>their v</i> Set $t = 0$ dep on 1 st M1
12	(b)	(ii)	M1 A1 2	Substitute $t=4$ oe
12	(c)	 	B1 B1 2	Inverted parabola implies symmetry about $t = 2$ through $(0,0)$ and $(4,0)$ (sight of correct vertex not necessary) Straight line through $(2, 0)$ and $(0, \text{their } 8)$ ($4, -\text{their } 8$)

Question		Answer	Marks	Rationale
13	(i)	$x \leq 20, y \leq 30$ $y \geq x$ $y \leq 2x$	B1 B1 B1 3	Condone $<$ for \leq and $>$ for \geq Ignore extras (including for instance, $x + y \leq 50$)
13	(ii)		B1 B1 B1 B1 B1 5	$x = 20$ $y = 30$ $y = x$ $y = 2x$ Shading Ignore any shading that may relate to 13(iii)
13	(iii)	<p>The line is $40x + 50y = 2000$</p>  <p>(20, 24) Total = 44 www</p>	B1 B1 B1 4	accept inequality soi (accept written in (ii)) Draw line (Correct line seen on graph is B2) Point soi Total (Total of 44 seen with nothing else is B2)

Question		Answer	Marks	Rationale
14	(i)	$\frac{dy}{dx} = 12x^2 - 10x$ <p>When $x = 1$, $m = 2$</p> $\Rightarrow \text{gradient of normal} = -\frac{1}{2}$ $\Rightarrow \text{Equation is } y - 0 = -\frac{1}{2}(x - 1)$ $\Rightarrow 2y + x = 1$	M1 A1 A1 M1 A1 5	Diffn and sub $x = 1$ www FT <i>their</i> numerical m soi Find eqn of line dep on M1 and <i>their</i> normal gradient oe (eg $y = -\frac{1}{2}x - \frac{1}{2}$)
14	(ii)	$-\frac{1}{2}(x - 1) = 4x^3 - 5x^2 + 1$ $\Rightarrow 1 - x = 8x^3 - 10x^2 + 2$ $\Rightarrow 8x^3 - 10x^2 + x + 1 = 0$	M1 A1 2	Substitute <i>their equation</i> At least 1 correct intermediate step seen, beware answer given
14	(iii)	For line: $y = -\frac{1}{2}\left(\frac{1}{4} - 1\right) = -\frac{1}{2} \times -\frac{1}{2} = \frac{1}{4}$ oe For curve: $y = 4\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right)^2 + 1 = \frac{1}{2} - \frac{5}{4} + 1 = \frac{1}{4}$	B1 B1 2	Substitute into <i>correct</i> line Substitute into curve

Question		Answer	Marks	Rationale
14	(iv)	$f(x) = 0 \text{ when } x = 1 \text{ and } \frac{1}{2}$ $\Rightarrow f(x) = (x-1)(2x-1)(4x+1)$ $\Rightarrow x = -\frac{1}{4}$ $y = \frac{5}{8}$ $\Rightarrow C \text{ is } \left(-\frac{1}{4}, \frac{5}{8}\right) \text{ www}$	M1 A1 A1 3	$f(x) = (x-1)(2x-1)(ax+b) \text{ oe}$ $\text{or } f(x) = (x-1) \left(x - \frac{1}{2}\right) (cx+d)$ <p>NB The working for this part may appear elsewhere but can only be credited if their final answer is seen here.</p>
		<p><i>Alternative method:</i></p> <p>Long division by $(x-1)$ and $(2x-1)$ or by one given factor plus attempt to factorise resulting quadratic or by $(2x^2-3x+1)$ M1 Giving third root A1 Answer A1</p>		Evidence of long division on cubic is correct first line of long division plus kx^2 in the quotient

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