OCR
Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
Isw	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
AG	Answer given
M1 dep	Method mark dependent on a previous method mark(s)
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for Additional Mathematics

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 can never be awarded.

B
Mark for a correct result or statement independent of Method marks.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$1-2(x-3)>4x$ $\Rightarrow a>bx$ or $-a<-bx$ Either $a=7$ or $b=6$ in either of above $\Rightarrow x<\frac{7}{6}$ $\left(\text{or } 1.17 \text{ or } 1.1\dot{6}\right)$	**M1** **A1** **A1**	Expand and collect soi www isw	Do not allow = anywhere even if final answer correct
				3		

Question			Answer	Marks	Guidance	
2			$\frac{dy}{dx}=3x^2-4x+2$ $\Rightarrow y=x^3-2x^2+2x\ (+c)$ **oe** Satisfied by (1, 3) $\Rightarrow 3=1-2+2+c$ $(\Rightarrow c=2)$ $\Rightarrow y=x^3-2x^2+2x+2$	**M1** **A1** **M1dep** **A1**	Int: At least 1 power increased by 1: Beware mult by x Three terms ignoring c Substitution Complete simplified equation	
				4		

Question			Answer	Marks	Guidance	
3			$3\sin x = 4\cos x \Rightarrow \tan x = \frac{4}{3}$ $\Rightarrow x = 53.1(3)$ and $x = 180 + 53.13 = 233(.13)$ Alternative: Square, use Pythagoras M1 $\Rightarrow \cos x = \pm 0.6$ or $\sin x = \pm 0.8$ A1(must include $\pm$) Gives 53.1 A1 Or 233 B1 only if no extra values in range	**M1** **A1** **A1** **B1**	For tan$x$ For $\frac{4}{3}$ One angle (53 not acceptable) ft Other angle B0 any extra values in range, ignore any outside range	Allow $\tan^{-1}\left(\frac{4}{3}\right) = k$ or $\tan^{-1}\left(\frac{3}{4}\right) = k$
				4		

Question			Answer	Marks	Guidance	
4	(i)		$8 - 4 + 2 - 6 = 0$ Alternative: Demonstration that $f(x) = (x-2)(x^2 + x + 3)$	**B1**	must be seen	i.e. powers evaluated
				1		
	(ii)		$f(x) = (x-2)(x^2 + x + 3)$ $D = b^2 - 4ac = 1 - 4 \times 3\ (= -11)\ (< 0)$ or $(x + 0.5)^2 + 2.75 \neq 0$ so only one root or no other roots	**M1** **A1** **M1** **A1**	Factorise: Any 2 correct terms of 3 term quadratic seen. For long division: first two terms For quad factor Numerical evidence must be seen on correct quadratic. Last statement must be seen. Condone reference to $(x - 2)$ being the root.	If quad factor is found in (i) then give credit in (ii) if seen in (ii) e.g. $\sqrt{-11}$ won't work
				4		

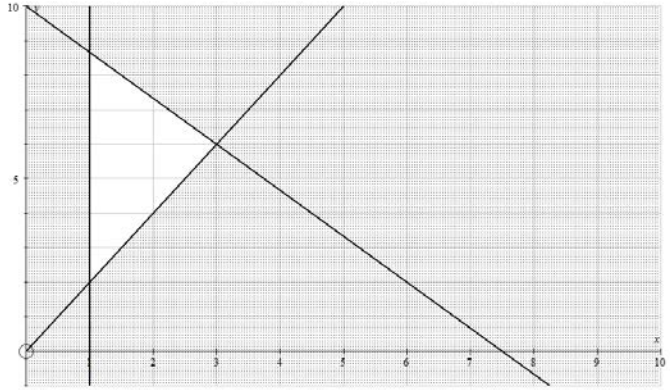
Question			Answer	Marks	Guidance	
5	(i)		11.5	B1	One number only seen or AB clearly identified	
				1		
	(ii)		Use 11.5, 15.5 and 20.5 $\cos B = \frac{11.5^2 + 15.5^2 - 20.5^2}{2\times 11.5\times 15.5}$ $(= -0.1339...)$ $\Rightarrow B = 97.7^0$	B1 M1 A1	Correct use of cosine rule on correct angle using values rounding to given values Answers rounding to 97.7	i.e. range [11.5,12.5]. [15.5,16.5],[19.5,20.5] Values must be consistent.
				3		

Question			Answer	Marks	Guidance	
6	(i)		$(\text{Distance for A:})\ \frac{3}{4}t^2$ $(\text{Distance for B: })\ t^2$ $\Rightarrow s = \frac{3}{4}(s \pm 100)$ or $s \pm 100 = \frac{3}{4}$s or $\frac{3}{4}t^2 \pm 100 = t^2$ $\Rightarrow s = 300$ or 400 or $t = 20$ $\Rightarrow$ B travels 400 m	B1 B1 M1 A1	soi; ignore 100 soi; ignore 100 Equating distances leading to one of the 6 forms www	SC4 www for trial and error giving correct answer.
				4		
	(ii)		Using $v^2 = u^2 + 2as$ $\Rightarrow v^2 = 2.2.400 = 1600$ $\Rightarrow v = 40$ m s^{-1}	M1 A1	And using a = 2 and *their* s from (i) www	Or complete and equivalent method. Allow missing u
				2		

Question			Answer	Marks	Guidance	
7	(i)		2	B1		
				1		
	(ii)		For PB: $PB = \sqrt{2^2 + 1.5^2} = 2.5$ $\Rightarrow \tan PBQ = \frac{2}{2.5} = 0.8$ $\Rightarrow \text{Angle PBQ} = 38.7^\circ$	M1 A1 M1 A1	 Using *their* PQ and PB Alternatively for the last two marks: Attempt to find QB and use it with sin, cos or sine rule or cosine rule	If PB is found in (i) then give credit in (ii) only if seen in (ii). n.b. $QB = \sqrt{10.25}$
				4		

Question			Answer	Marks	Guidance	
8	(i)		$(1+\delta)^3 = 1^3 + 3.1^2\delta + 3.1\delta^2 + \delta^3$ $= 1 + 3\delta + 3\delta^2 + \delta^3$	B1 B1	Unsimplified expansion soi	Can be by expansion
				2		
	(ii)		Because, if δ is small, then (terms in) $(3)\delta^2$ and δ^3 are very small and can be ignored	B1	"ignored" or similar must be seen	e.g. neglected, eliminated
				1		
	(iii)		$(1+\delta)^3 - 0.9(1+\delta) - 0.206 \quad (=0)$ $\Rightarrow 1 + 3\delta - 0.9(1+\delta) - 0.206 \quad (=0)$ $\Rightarrow 2.1\delta = 0.106$ $\Rightarrow \delta = 0.05.....$ $\Rightarrow x = 1.05...$	M1 M1dep A1 A1	Sub Using result of (ii) 3sf or better	
				4		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{dy}{dx} = 3x^2 - 6x - 3$ When $x = 3, \frac{dy}{dx} = 6$ $\Rightarrow$ Equation of tangent is $y + 5 = 6(x - 3)$ **oe** $\Rightarrow y = 6x - 23$ **oe**	**M1** **A1** **M1dep** **A1**	Diffn. At least one power reduced by 1. Beware division by x Any valid form using *their* gradient and (3, –5). **oe** only 3 terms	Ignore +c
				4		
	(ii)		$\frac{dy}{dx} = 3x^2 - 6x - 3 = 6$ $\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x - 3)(x + 1) = 0$ $\Rightarrow$ Q is where $x = -1, y = 3$	**M1** **A1** **A1**	Equating *their* gradient fn and *their* 6 Correct factorisation www cao www	Ignore (3, –5) as a possible answer. SC3 if $\frac{dy}{dx} = x^2 - 2x - 1 \Rightarrow$ g = 2 in (i) and Q is correct.
				3		

Question			Answer	Marks	Guidance	
10	(i)			**B1** **B1** **B1** **B1** **B1**	One line 2nd line 3rd line Shading $x \leq 1$ Other shading. Allow ft if gradients of lines are the same sign as the correct lines.	Allow one small square tolerance on each axis
				5		
	(ii)		Max value at intersection which is (3, 6) =45	**B1** **B1**	soi	e.g. 45 gets 2
				2		

Section B

Question			Answer	Marks	Guidance	
11	**(i)**		(OP =) $100 - 20t$ (OQ =) $400 - 25t$	**B1** **B1**	isw isw	Ignore labels
				2		
	(ii)		$\left(x^2\right) = \left(100 - 20t\right)^2 + \left(400 - 25t\right)^2$ $x^2 = 170000 - 24000t + 1025t^2$	**M1** **A1** **A1**	Use of Pythagoras on *their* expressions. Soi ignore lack of x^2 Final answer must include x^2	Condone use of $20t - 100$ etc for full marks
				3		
	(iii)		$\frac{\mathrm{d}}{\mathrm{d}t}\left(x^2\right) = -24000 + 2050t$ $= 0$ when $t = \frac{24000}{2050}$ $\left(= \frac{480}{41}\right) = \left(11.7\right)$ **oe** Then $x^2 = 29512$ $\Rightarrow x = 172\,(\mathrm{m})$	**M1** **A1** **M1dep** **A1** **M1dep** **A1**	Diffn of *their* fn : reduction of power in at least one term Correct numerical expression isw Set = 0 and attempt to solve Allow correct answer even if premature division in (i) Substitute *their t*(providing $t > 0$). Dep on both M	Ignore incorrect constant from (ii) Ignore notation on lhs SC 1 for $b + 2ct$
				6		
	(iv)		Car takes 5 secs to reach O Train takes 16 secs	**B1**	Numerical evidence for both required	Accept other valid explanations
				1		

Question			Answer	Marks	Guidance	
12	(i)		$x + 3y = k$ or $y = -\frac{1}{3}x + c$ or $\frac{y-b}{x-a} = -\frac{1}{3}$ substitute (8, 3) gives $x + 3y = 17$ **oe**	**M1** **M1dep** **A1**	3 term equation isw	$k = 17$ or $c = \frac{17}{3}$ $y = -\frac{1}{3}x + \frac{17}{3}$
				3		
	(ii)		Solve *their* L_2 with $y = 3x - 1$ simultaneously: $x = 2$, $y = 5$	**M1** **A1** **A1**	Must lead to a value for x or y	SC3 Checking points and finding that (2, 5) lies on both
				3		
	(iii)		$d^2 = (8-2)^2 + (3-5)^2 \quad (= 40)$ $\Rightarrow d = \sqrt{40} \quad (= 2\sqrt{10} = 6.32)$	**M1** **A1**	Application of Pythagoras	
				2		
	(iv)		$(x-8)^2 + (y-3)^2 = 40$	**B1**	FT from (iii) Allow 6.32^2 oe	
				1		
	(v)		The point is on the other end of the diameter: (2, 5) to (8, 3) is $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $\Rightarrow (14,1)$ $3x - y = c$ satisfied by (14, 1) $\Rightarrow 3x - y = 41$ **oe**	**M1** **A1** **A1**	Using (8, 3) and *their* Q from (ii). Only 3 terms	Alternatively: $\frac{2+x}{2} = 8, \quad \frac{5+y}{2} = 3$ M1 $\Rightarrow x = 14, \; y = 1$ A1
				3		

Question			Answer	Marks	Guidance	
13	(i)		$\frac{540}{x}, \frac{540}{x+75}$ **oe**	**B1** **B1**	Condone $\frac{5.4}{x}, \frac{5.40}{x+0.75}$ or $\frac{5.40}{x+75}$	Ignore any labels. Allow $n \leqslant \frac{540}{x}$ etc
				2		
	(ii)		$\Rightarrow \frac{540}{x} = \frac{540}{x+75} + 5$ **oe**	**M1**	For forming 3 term eqn using *their* terms from (i) Condone -5	May start again
				A1	Correct eqn	
				M1	Clear both fractions. Eqn must have 3 terms with x and $x \pm 75$ involved in denominator for 2 terms	
			$\Rightarrow 540(x+75) = 540x + 5x(x+75)$ **oe**	**A1**		
			$\Rightarrow (540 \times 75 = 5x(x+75))$			
			$\Rightarrow x^2 + 75x - 8100 = 0$	**A1**	AG. At least 1 intermediate step must be seen	Any wrong algebra gets final A0
				5		
	(iii)		$x^2 + 75x - 8100 = 0$	**M1**	Solving given quadratic by factorisation that would lead to 2 terms correct when expanded soi Or correct formula soi	
			$\Rightarrow (x-60)(x+135) = 0$	**A1**	Correct factorisation or correct substitution	
			$\Rightarrow x = 60$	**A1**		Ignore -135
			$\Rightarrow x + 75 (= 135)$ or $60 + 75$	**A1**	soi by final answer	
			Buns 60p, loaf of bread 135p **oe**	**A1**	cao www - units must be given	Correct answer only full marks
				5		

Question			Answer	Marks	Guidance	
14	(i)		$\Rightarrow 7=(-3)^3+(-3)^2a-3b+1$ **oe** and $3=1+a+b+1$ **oe** $\Rightarrow \quad (9a-3b=33 \text{ and } a+b=1\ )$ $\Rightarrow a=3,\ b=-2$	**B1** **B1** **M1** **A1**	1st equation 2nd equation Solve *their* eqns leading to either *a* or *b* Both **AG**	Need not be simplified for either Need to see at least one intermediate step
				4		
	(ii)		Midpoint is (–1, 5) Show (–1, 5) lies on curve.	**B1** **B1**	Must see –1 + 3 + 2 + 1 = 5	i.e. powers must be evaluated
				2		
	(iii)		$A_1=\pm(\text{Area under curve}-\text{area under line})$ or $A_2=\pm(\text{Area under line}-\text{area under curve})$ Area under curve $=\int(x^3+3x^2-2x+1)\,dx=\frac{x^4}{4}+x^3-x^2+x(+c)$ $A_1=\left(\left(\frac{1}{4}-1-1-1\right)-\left(\frac{81}{4}-27-9-3\right)\right)-12$ $=\left(-\frac{11}{4}--\frac{75}{4}\right)-12=16-12=4$ $A_2=8-\left(\left(\frac{1}{4}+1-1+1\right)-\left(\frac{1}{4}-1-1-1\right)\right)=8-4=4$	**B1** **M1** **A1** **M1dep** **A1** **A1**	For sight of one attempt to find a difference of areas For integration, ignore limits Integration correct Correct limits for one curve integral (For A_1, –3 to –1 and for A_2, –1 to 1) For A_1 www For A_2 www	 At least 3 powers increased by 1. Watch for multiplication by *x* Could be wrong way round but must be subtracted n.b. an answer of –4 should be explained for credit of A1
				6		

Question	Answer	Marks	Guidance	
	Alternative 1: if subtraction is before integration.	**B1**	For subtracting *their* $y = 4 - x$ from curve	Could be subtracted in either order
	$(A_1) = \int \left(x^3 + 3x^2 - 2x + 1 - (4 - x)\right) dx = \frac{x^4}{4} + x^3 - \frac{x^2}{2} - 3x (+c)$	**M1**	For either integration, ignore limits	
	$= \left(\left(\frac{7}{4}\right) - \left(-\frac{9}{4}\right)\right) = 4$	**A1**	Either integration correct	
		M1dep	Correct limits for one curve integral (For A_1, -3 to -1 and for A_2, -1 to 1)	Could be wrong way round but must be subtracted.
	$(A_2) = \int \left((4 - x) - (x^3 + 3x^2 - 2x + 1)\right) dx = -\frac{x^4}{4} - x^3 + \frac{x^2}{2} + 3x (+c)$	**A1**	For A_1 www	n.b. an answer of -4 should be explained for credit of A1
	$= \left(\left(\frac{9}{4}\right) - \left(-\frac{7}{4}\right)\right) = 4$	**A1**	For A_2 www	
	Alternative 2	**B1**	For subtracting *their* $y = 4 - x$ from curve	
	$y = \left(x^3 + 3x^2 - 2x + 1\right) - (4 - x) = x^3 + 3x^2 - x - 3$	**M1**	Writing as a function of $(x + 1)$	
	$y = (x + 1)^3 - 4(x + 1)$	**A1**		
	This is an odd function relative to $x = -1$. The function therefore has 180^0 rotational symmetry about $(-1, 0)$	**M1**	Understanding of odd function	
	So $A_1 = A_2$	**A1**	Rotational symmetry	
		A1	Conclusion	

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