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Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise;

e The abbreviation **ft** implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$2-x<1+3(x-2)$ $\Rightarrow 2<4x-5$ $\Rightarrow 4x>7$ $\Rightarrow x>\frac{7}{4}$	**B1** **B1** **B1**	Remove brackets giving rhs $1 + 3x – 6$ or better **Ft** Result in the form $ax>b$ **oe**	
				[3]		

Question			Answer	Marks	Guidance	
2			$\frac{dy}{dx}=2+2x-3x^2$ $\Rightarrow (y=)2x+x^2-x^3(+c)$ Through (2, 3) $\Rightarrow 3=4+4-8+c$ $\Rightarrow c=3$ $\Rightarrow y=2x+x^2-x^3+3$	**M1** **A1** **M1** **A1**	Attempt to integrate – at least two powers increased by 1 Coefficients unsimplified Dep. Substitute in *their* function Equation must be given	Beware of mult by x
				[4]		

Question			Answer	Marks	Guidance	
3	(i)		(x =) -3, 1	B1	Both, by any means	Allow (-3, 0) and (1, 0)
				[1]		
	(ii)		The line is (y =) $3x + 3$ The solution is 3, -2	B1 B1 B1	Equation (soi by plot) Correctly plotted Dep on previous 2	Algebraic soln not acceptable. Allow (3, 12) and (-2, -3)
				[3]		

Question			Answer	Marks	Guidance	
4			$\cos^2\theta = 1-\sin^2\theta = 1-\frac{1}{25}$ $\Rightarrow \cos\theta = \sqrt{\frac{24}{25}}$ or $\frac{2}{5}\sqrt{6}$	M1 A1	Use of Pythagoras Isw allow $\sqrt{0.96}$	
			$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{5}\times\frac{5}{2\sqrt{6}}$ $= \frac{1}{\sqrt{24}}$ or $\frac{1}{2\sqrt{6}}$ or $\frac{1}{12}\sqrt{6}$	M1 A1	Use of tan ratio Isw allow $\frac{1}{5\sqrt{0.96}}$	M0 if approximate values used
				[4]		
			Alternatively: Find third side of triangle $= \sqrt{24}$ giving $\cos\theta = \sqrt{\frac{24}{25}}$ and $\tan\theta = \sqrt{\frac{1}{24}}$	 M1 A1isw A1 isw A1 isw		

Question			Answer	Marks	Guidance	
5			$\int_0^3 \left(6x^2 - 2x^3\right) dx$	M1	Integration – ignore limits	i.e. both powers increased by 1
			$= 2x^3 - \frac{x^4}{2}$	A1		Allow unsimplified
			$\Rightarrow A = \left[2x^3 - \frac{x^4}{2}\right]_0^3$			
			$= \left(54 - 40\frac{1}{2}\right) - 0$	M1	Dep. Substitution of $x = 3$ (and $x = 0$ soi). Any other limits M0	
			$= 13\frac{1}{2}$	A1		SC Answer only or www seen B4
				[4]		

Question			Answer	Marks	Guidance	
6	(i)		(i) × 3: $9x + 12y = 54$	M1	Making a coefficient the same	Alternatively soln by substitution
			(ii) × 4: $28x - 12y = 20$ Add: $37x = 74$	M1	Elimination	
			$\Rightarrow x = 2$	A1		SC Answer only or www seen B4
			$\Rightarrow y = 3$	A1		
				[4]		
	(ii)		Sketch to show two lines, one +ve gradient and one –ve,	B1	Two lines	
			intersecting at *their* point from (i)	B1	Dep. *Their* intersection	
				[2]		

Question			Answer	Marks	Guidance	
7	(i)		$7x-9=x^2+2x-5$ $\Rightarrow x^2-5x+4=0$ $\Rightarrow (x-4)(x-1)=0$ $\Rightarrow x=1,4$ $\Rightarrow y=-2,19$ $\Rightarrow (1,-2),(4,19)$	**M1** **A1** **A1** **A1**	Equate Correct quadratic Both x-values (or both y-values or one pair) Both coordinates	Alt: Make x subject and substitute to give $y^2-17y-38=0$ Allow $x = 1$, $y = -2$ and $x = 4$, $y = 19$ SC. Answer (i.e. both pairs) only or www B4
				[4]		
	(ii)		$\left(\frac{dy}{dx}=\right)\ 2x+2$ At $(1,-2)$, $\frac{dy}{dx}=2+2=4$ At $(4,19)$, $\frac{dy}{dx}=8+2=10$ Grad normals = $-\frac{1}{4}$ and $-\frac{1}{10}$ $\neq 7$ so no. **oe**	**M1** **A1** **A1**	Diffn Both values Correct comparison	Alternative: Diffn M1 One value seen and correct numeric comparison A1
				[3]		

Question			Answer	Marks	Guidance	
8	(i)		$\frac{x+a}{x}+\frac{x-2}{4}=0$ $\Rightarrow 4x+4a+x^2-2x=0$ $\Rightarrow x^2+2x=-4a$ $\Rightarrow x^2+2x+1=1-4a$ $\Rightarrow (x+1)^2=1-4a$	**M1** **M1** **A1**	Clear fractions on lhs Collection of terms to a 3 term quadratic and attempt to complete the square Correct final form	"Attempt" means make lhs include $x^2+2px+p^2$
				[3]		
	(ii)		(Roots if) *their* $q \geqslant 0$ $\Rightarrow a \leqslant \frac{1}{4}$	**M1** **A1**	Soi. Allow use of >. **ft** *their q* . correct inequality.	Allow = here only if ans is correct. Allow expansion of quadratic and use of discriminant
				[2]		
	(iii)		$(x+1)^2=5$ $\Rightarrow x=-1\pm\sqrt{5}$	**M1** **A1**	Substitute to obtain quadratic in form $(x+p)^2=n$ Both required isw	Allow use of formula
				[2]		

Question			Answer	Marks	Guidance	
9	(a)	(i)	$\left(\frac{4}{5}\right)^{10} = 0.107(4)$	M1 A1	Correct power and *p* Awrt isw	One term only
				[2]		
		(ii)	$\binom{10}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6 = 210\times 0.00041943$ $= 0.088(08)$	M1 A1 A1	Includes correct powers and a coefficient 210 soi Awrt isw	
				[3]		
	(b)		Fixed number of trials Each trial has two outcomes Fixed probability for success Independent trials	B1 B1	Any one correct Another correct	Ignore incorrect answers or other answers
				[2]		

Question			Answer	Marks	Guidance	
10	(i)		Because AP = XP = GP (= radius)	B1		
				[1]		
	(ii)		Angle GAX is angle in semicircle and since BA is vertical, XA must be horizontal.	B1		Accept any valid method
				[1]		
	(iii)		Finding AG = 120cos40 (= 91.93) **oe** Finding depth of Y below G =80cos40 (=61.28) Ht of Y above ground = 200 – AG – depth of Y below G =46.8 cm	M1 M1 M1 A1	 Need not be numeric	Distance of Y below AX = 200cos40 M1(correct triangle) M1 (correct ratio) = 153.2 cm So height above ground = 200 – 153.2 M1 = 46.8 cm A1
				[4]		

Section B

Question			Answer	Marks	Guidance	
11	(i)		$x^2+(y-3)^2=9$ $\Rightarrow x^2+y^2-6y+9=9$ $\Rightarrow x^2+y^2-6y=0$ $(k=6)$	**M1** **A1**	 Isw	
				[2]		
	(ii)		They meet when $x^2+(mx-2)^2-6(mx-2)=0$ $\Rightarrow (1+m^2)x^2-10mx+16=0$ **oe** Tangent if coincident roots $\Rightarrow (-10m)^2=4.16(1+m^2)$ **oe** $\Rightarrow 36m^2=64$ $\Rightarrow m=\pm\frac{4}{3}$	**M1** **A1** **M1** **A1** **A1** **A1**	Substitute line into *their* curve **ft** Allow bracket expanded Dep. Attempt to find coincident roots using "b^2-4ac"	Or substitute for x to give quadratic in y: $y^2(1+m^2)+y(4-6m^2)+4=0$ $\Rightarrow 36m^4-64m^2=0$
				[6]		
	(iii)		In triangle PCA, PC = 5 CA = 3 By Pythagoras: PA = PB = 4	**B1** **B1** **M1** **A1**	soi soi Both	
				[4]		

Question			Answer	Marks	Guidance	
12	(i)		$p = 2y + 6x + AE + DE$ $AE = DE = 5x$ Giving $p = 2y + 6x + 5x + 5x$ $p = 2y + 16x$	**M1** **B1** **A1**	Adding www soi www **AG** algebra must be correct	N./B. $\sqrt{3^2x+4^2x} = \sqrt{25x} = 5x$ or $\sqrt{3x^2+4x^2} = \sqrt{25x^2} = 5x$ could earn M1 B0 A0
				[3]		
	(ii)		$A = 6xy + 12x^2$ $= 3x(96 - 16x) + 12x^2$ $= 288x - 36x^2$	**M1** **M1** **A1**	Calculate the area Substitute correct expression for y **AG**	
				[3]		
	(iii)		$A = 288x - 36x^2$ $\Rightarrow \frac{dA}{dx} = 288 - 72x$ $= 0$ when $x = \frac{288}{72} = 4$ $\Rightarrow A = 288 \times 4 - 36 \times 16 = 576\text{cm}^2$ $\Rightarrow y = \frac{96-64}{2} = 16$ Alternatively: $A = 288x - 36x^2 = 36\left(8x - x^2\right) = 36\left(16 - \left(16 - 8x + x^2\right)\right)$ **M1 A1** $= 36\left(16 - \left(x - 4\right)^2\right)$ which has its greatest value when $x = 4$ **M1 A1** $\Rightarrow A = 36 \times 16 = 576$ **A1** $y = 48 - 8x = 16$ **A1**	**M1** **A1** **M1** **A1** **A1** **A1**	Diffn - reduce each power by 1 Set = 0 x area y	SC Graph of fn goes through (0,0) and (8,0) so being quadratic means max value at x = 4 B4 Area B1 y value B1 Other symmetrical points may be used.
				[6]		

Question			Answer	Marks	Guidance	
13	(i)		$AC^2 = 25 + 4 - 2.2.5\cos 40$ $= 13.68$ $\Rightarrow$ AC = 3.70 km	**M1** **A1** **A1** **A1**	Cos formula Correct subs soi AC^2 soi cao	
				[4]		
	(ii)		$\frac{5}{3}$ hrs $(=100$ mins$)$ $\frac{3.7}{2}$ hrs $(=111$ mins$)$ $111 - 100 = 11$ mins	**B1** **B1** **B1**	 www **AG**	J = 100 mins B = 111 mins
				[3]		
	(iii)		$\frac{\sin\theta}{3} = \frac{\sin 40}{2}$ **oe** $\Rightarrow \sin\theta = \frac{3}{2}\sin 40 \ (= 0.9642)$ $\Rightarrow \theta = 74.6^0$ Bearing 345^0 or 105.4^0 Bearing 015^0	**M1** **A1** **A1** **A1** **A1**	Sin rule with denominators in proportion 3:2 Soi One angle awrt Correct bearing awrt 2nd angle plus bearing A1awrt (Allow 15º) awrt	Solution using cosine rule acceptable. Alternatively 2nd angle Then both bearings
				[5]		

Question			Answer	Marks	Guidance	
14	(a)	(i)	$s = \frac{1}{2} 2t^2 \left(= t^2\right)$	B1		
				[1]		
		(ii)	90 km h^{-1} = 25 m s^{-1} or 2ms^{-2} = 25920 km hr^{-2}	B1	Units must be given - others are possible	Beware mixing of units which could give 12.5
			$v = 2t \Rightarrow 25 = 2t$	M1	Application of $v = u + at$ with consistent units	
			$\Rightarrow t = 12.5$ secs	A1	Units must be given	
				[3]		
	(b)	(i)	When t = 10 s_P = 100 For Q: $a = 1 + kt$	B1 M1	Seen anywhere Integrating wrt t - both powers increased by 1	
			$\Rightarrow v = t + \frac{kt^2}{2}$	A1	Must not include c.	
			$\Rightarrow s_Q = \frac{t^2}{2} + \frac{kt^3}{6}$	A1	Must not include $ct + d$.	
			At $t = 10$ $s_Q = 50 + \frac{1000k}{6} = 100$			
			$\Rightarrow k = \frac{300}{1000} = 0.3$	A1		
				[5]		
		(ii)	When t = 12.5 $v_Q = t + \frac{kt^2}{2} = 12.5 + \frac{0.3 \times 12.5^2}{2}$	M1	Inserting *their t* and *their k* into velocity eqn	
			$= 35.94$ m s^{-1}	A1		
			$= 35.94 \times \frac{60 \times 60}{1000} = 129.4$ km h	A1	**AG**	
				[3]		

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