OCR
Oxford Cambridge and RSA

Level 3 Certificate

Mathematical Techniques and Applications for Engineers

Unit **H865/01** Component 1

OCR Level 3 Certificate

Mark Schemes for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations in scoris

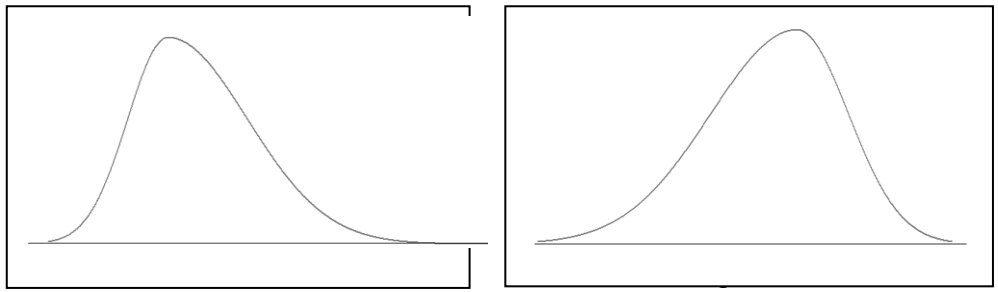
The following annotations are available:

BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓	correct response
✗	incorrect response
ecf	error carried forward
BOD	Benefit of doubt

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
1	$6x - (8x + 5) = 6x - 8x - 5 = -2x - 5$ $-2x$ -5	**[1]** **[1]**	
2	$x^2 + 5x - 14 = (x + 7)(x - 2)$ $(x + 7)$ $(x - 2)$	**[1]** **[1]**	
3	$[(x + 4)/8] - (x - 5)/2 = (x + 4 - 4x + 20)/8$ or $((2x + 8) - (8x - 40))/16$ or $(x + 4)/8 - (4x - 20)/8$ or $(2x + 8)/16 - (8x - 40)/16$ $= (-3x + 24)/8$ or $(-6x + 48)/16$ $(-3x + 24)$ or $(-6x + 48)$ 8 or 16	**[1]** **[1]**	If denominator is subsequently removed from final answer i.e. final answer $= (-3x + 24)$ or $(-6x + 48)$ award one mark only. Also accept $-3x/8 + 3$ for 2 marks
4	$6(3x - 5) = 2(4x - 7)$ $18x - 30 = 8x - 14$ $10x = 16$ $x = 1.6$	**[1]** **[1]**	One mark for $18x - 30 = 8x - 14$ or $10x = 16$ or $-10x = -16$ CAO

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section A					
5	**(a)**		$\pi/3$ radians $= \pi$ x $(360/6\ \pi) = 60^o$	**[1]**	Award one mark for correct numerical results with or without the unit.
5	**(b)**		$120^o = 120$ x $(2\ \pi/360) = \frac{2}{3}\pi = 2.0944$ radians.	**[1]**	Accept $\frac{2}{3}\pi$ or answers between 2.05 and 2.15 radians.
6			Sine wave.	**[1]**	Accept y=sin(x) or just y=sin or sin
7			In the triangle ABC angle A = 30^o and length of side 'a' = 100 mm. Angle C = 90 – 30 = 60^0 .Side c = a/tan A = 100/tan 30^0 = 173.2 mm. --- Alternative solution Angle C = 90 – 30 = 60^0 100/sin30 = c/sin60 c = 100sin60/sin30 = 173.2 mm	**[1]** **[1]** **---** **or** **[1]** **[1]**	Accept answers between 173 and 173.5 mm. Unit not required for full marks.
8			Given cosine x =3/5.  By observation the right-angled triangle will have sides of 3, 4 and 5 units long. So sine x = 4/5 or 0.8	**[1]** **[1]**	One mark for triangle drawn with correct values. CAO.

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
9	Given $y = 3\cos x + 6x^3$ So $dy/dx = -3\sin x + 18x^2$ $-3\sin x$ $+18x^2$	 **[1]** **[1]**	Allow $-3\sin x + 18x^2$ Allow $3(-)\sin x + 18x^2$
10	Given $y = \sin x + (1/x) = \sin x + x^{-1}$ Then $dy/dx = \cos x - x^{-2} = \cos x - (1/x^2)$ $\cos x$ $-(1/x^2)$ or $-x^{-2}$	 **[1]** **[1]**	Allow $\cos(-)x - (1/x^2)$
11	$\int 2\sin 5x\ dx = -0.4\cos 5x + C$ $-0.4\cos 5x$ or $-2/5(\cos 5x)$ $+C$	 **[1]** **[1]**	
12	$\int_2^5 4x^3\ dx = [x^4]_2^5 = 5^4 - 2^4 = 625 - 16 = 609$ $[x^4]_2^5$ $5^4 - 2^4$ 609	 **[1]** **[1]** **[1]**	Do not accept error carried forward if incorrect integration is used.

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
13	In a statistical distribution, the curve appears distorted either to the left or to the right. 	**[2]**	Diagram 1 mark Explanation 1 mark Award 2 marks for a clearly annotated diagram
14	Given 6 9 7 11 8 10 6 Mode = 6 Values arranged in order of magnitude: 6 6 7 <u>8</u> 9 10 11 Median point = 8	**[1]** **[1]**	
15	P(A) = probability that event A happens. . P(B) = probability that event B happens. .	**[1]** **[1]**	Accept similar explanations but these MUST include reference 'Probability' or 'Likelihood' or 'possibility' and must mention (event) A (and B). eg 'likelihood of (event) A (happening)' Do not accept eg: 'A will happen' or 'A happens' etc
	Total	**[30]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section B					
1	**(a)**		Given $W = (mv^2) /2$. Substitute m = 5 and v = 10. Then $W = (5 \times 10^2) /2$ $= 250$	**[1]**	
1	**(b)**		Given that $A = B(1 + 2CD)$ Open the brackets then $A = B + 2BCD$ Then $2BCD = A – B$ So $C = (A – B)/2BD$ --- Alternative solution $A/B = (1+2CD)$ $A/B – 1 = 2CD$ $C=(A/B – 1)/2D$	**[1]** **[1]** **[1]** **------** **or** **[1]** **[1]** **[1]**	Accept any other correct method.
1	**(c)**		Given $v^2 = u^2 +2as$ Subtract both sides by 2as Then $u^2 = v^2 – 2as$ Square root both sides So $u = \sqrt{(v^2 – 2as)}$	**[1]** **[1]**	Accept any other correct method.

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section B					
1	**(d)**		Given A = B/(B + 2)		
			Multiply both sides by (B + 2) then AB + 2A = B	**[1]**	Accept any other correct method.
			Subtract both sides by B then AB – B = – 2A	**[1]**	
			Then B(A – 1) = – 2A	**[1]**	
			So B = – 2A/(A – 1) or 2A/(1 – A)	**[1]**	
			Total	**[10]**	

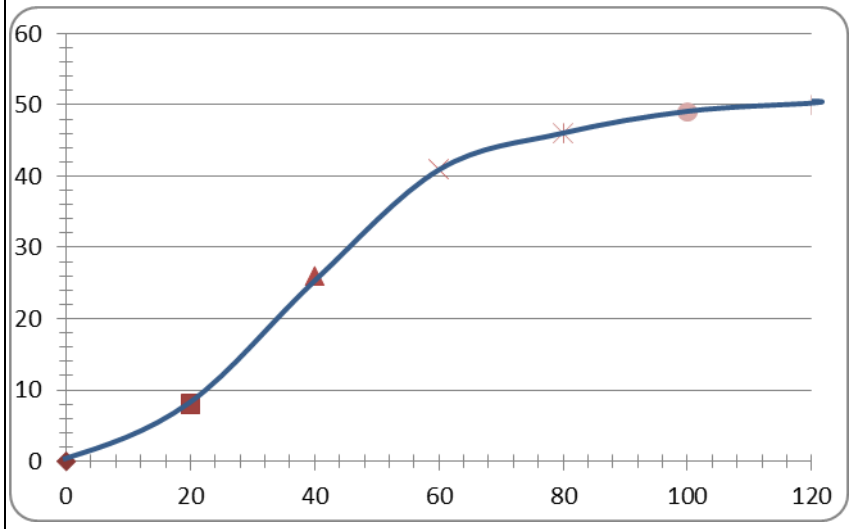
Question			Expected Answer	Mark	Rationale/Additional Guidance
2	(a)		Area of triangle A = ½bh So perpendicular height h = 2A/b = (2 x 1000)/40 = 50 mm.	[1] [1]	Unit not required for full marks
2	(b)		Given x = 8 m, y = 10 m and z = 12 m. Area of triangle A = $\sqrt{}$ [s(s –x)(s – y)(s – z)] where s = (x + y + z)/2 So s = (8 + 10 + 12)/2 = 15 Then Area A = $\sqrt{}$[15(15 – 8)(15 – 10)(15 – 12) = $\sqrt{}$1575 = 39.69 m^2 --- Alternative solutions $x^2 = y^2 + z^2 - 2yzcosX$ $cosX=(x^2 - y^2 - z^2)/(-2yz)$ $cosX=(8^2 - 10^2 - 12^2)/(-2\times 10 \times 12) = 0.75$ $X = 41.41^o$ Area = (10 × 12 × sin 41.41)/2 =39.69 m^2 $y^2 = x^2 + z^2 - 2xzcosY$ $cosY=(y^2 - x^2 - z^2)/(-2xz)$ $cosY=(10^2 - 8^2 - 12^2)/(-2\times 8 \times 12) = 0.56$ Y = 55.77 Area = (8 × 12 × sin 55.77)/2 =39.69 m^2	[1] [1] [1] [1] [1] --- or [1] [1] [1] [1] [1] or [1] [1] [1] [1] [1]	BEWARE ½(8 × 10) = 40 (no marks for this) Accept answers between 39 m^2 and 40 m^2 provided correct working is seen Unit not required for full marks

Question			Expected Answer	Mark	Rationale/Additional Guidance
			$z^2 = x^2 + y^2 - 2xycosZ$ $cosZ=(z^2 - x^2 - y^2)/ (-2xy)$ $cosZ=(12^2 - 8^2 - 10^2)/ (-2\times 8 \times 10) = 0.125$ $Z= 82.82^o$ Area = $(8 \times 10 \times \sin 82.82)/2$ $=39.69\ m^2$	**or** **[1]** **[1]** **[1]** **[1]** **[1]**	
2	**(c)**		Given angle B = 30^0, side a = 12 m and side c = 16 m. Area of triangle A = ½ac sin B $= ½ \times 12 \times 16 \sin 30^0$ $= 96 \times 0.5$ $= 48\ m^2$	**[1]** **[1]** **[1]**	Unit not required for full marks.
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
3	(a)		Given $s = 60(20e^{-t/20} + t - 20)$. Velocity = $ds/dt = 60(-e^{-t/20} + 1)$ Acceleration = $dv/dt = d^2s/dt^2 = 60(e^{-t/20}/20)$ When t = 10s Acceleration = $3\ (e^{-10/20})$ $= 1.82\ ms^{-2}$	[1] [1] [1] [1]	Unit not required for full marks
3	(b)		Given $S = 2\pi r^2 + 400/r = 2\pi r^2 + 400r^{-1}$ $ds/dr = 4\pi r - 400r^{-2} = 4\pi r - 400/r^2$ Surface area is a minimum when ds/dr = 0 So $4\pi r - 400/r^2 = 0$ $4\pi r = 400/r^2$ $4\pi r^3 = 400$ $r^3 = 400/4\pi$ So r = 3.17	[1] [1] [1] [1] [1] [1]	Allow solutions by trial and error (4 marks) for answers between 3.0 and 3.3 (2 marks)
			Total	[10]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
4	(a)	(i)		[1]	

Time (minutes)	Tally	Cumulative frequency (f)
$0 < t \leq 20$	8	8
$20 < t \leq 40$	18	26
$40 < t \leq 60$	15	41
$60 < t \leq 80$	5	46
$80 < t \leq 100$	3	49
$100 < t \leq 120$	1	50

Question			Expected Answer	Mark	Rationale/Additional Guidance
4	(a)	(ii)	Cf  Time	[3]	Consider ECF provided there is an obvious upward trend of values up to about 50 in the table. Award one mark for six correctly plotted points Award one mark for both correct axes. Award one mark for lines or curves joining points.
4	**(a)**	**(iii)**	median number = 39	**[1]**	Accept answers between 36 and 42
4	**(a)**	**(iv)**	60^{th} percentile = 45	**[1]**	Accept answers between 43 and 47
4	**(b)**		$\Sigma x = 27 + 35 + 23 + 40 + 35 = 160$ Mean = $\Sigma x/n = 160/5 = 32$ $\Sigma x^2 = 27^2 + 35^2 + 23^2 + 40^2 + 35^2 = 5308$ Standard deviation = $\sqrt{\{(\Sigma x^2)/n) - ([\Sigma x]/n)^2\}}$ $= \sqrt{\{ (5380/5) - (160/5)^2\}}$ $= 6.13$	**[1]** **[1]** **[1]** **[1]**	Accept answers between 6 and 6.5
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
5	(a)	(i)	Given $v = u + at$. $v = 60$ when $t = 10$ and $v = 30$ when $t = 4$. So $60 = u + 10a$ Equation 1 And $30 = u + 4a$ Equation 2 Subtract eqn 2 from eqn 1 Then $30 = 6a$ So $a = 30/6 = 5$ Substitute $a = 5$ into eqn 1 So $60 = u + (10 \times 5)$ Then $u = 60 – 50 = 10$	[1] [1] [1] [1] [1] [1]	
5	(a)	(ii)	When $t = 6$ use equation $v = 10 + 5t$ $v = 10 + (5 \times 6)$ $= 40$	[1]	
5	(b)		Given $3x^2 + 14x + 8 = 0$. By inspection $3x^2 = (3x)(x)$ And $8 = (1)(8)$ or $(2)(4)$ By factorisation $(3x + 2)(x + 4) = 0$ So $(3x + 2) = 0$ or $(x + 4) = 0$ When $3x + 2 = 0$ Then $3x = – 2$ so $x = – 2/3$ or 0.666… or 0.67 Likewise when $(x + 4) = 0$ Then $x = – 4$	[1] [1] [1]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
			Alternative method	**or**	
			$x = (–b±\sqrt{(b^2-4ac)})/(2a)$	**[1]**	
			$x = (–14±\sqrt{(14^2-4×3×8)})/(2×3)$		
			$x = (–14±\sqrt{(14^2-4×3×8)})/(2×3)$		
			$x = (–14±\sqrt{(100)})/6$		
			$x = (–14±10)/6$		
			x= –4/6 or– 2/3 or 0.666… or 0.67	**[1]**	
			x = –24/6 or –4	**[1]**	
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
6	(a)		In a triangle ABC. , AB = 5 m, AC = 8 m and angle ABC = 42^o. Let angle ABC be angle B. Let angle ACB be angle C. Sine Rule: AC/ sin B = AB/ sin C so sin C = (AB sin B)/AC Then sin C = (5 sin 42^o)/8 = 0.4182 So C = $\sin^{-1}$ 0.4182 = 24.72^o So Angle ACB is 24.72^o	**[1]** **[1]** **[1]** **[1]**	Accept answers between 24 and 25^0.
6	(b)		In a triangle ABC. Side a = 40 mm, side b = 70 mm and angle C = 50^0 Cosine Rule: Cos C = $(a^2 + b^2 – c^2)/2ab$ So $c^2 = a^2 + b^2 – 2ab$ Cos C = $(40^2 + 70^2 – [2 \times 40 \times 70 \cos 50^0])$ = 1600 + 4900 – [5600 x 0.6428] = 1600 + 4900 – 3600 = 2900 So c = √2900 = 53.85 mm.	**[1]** **[1]** **[1]** **[1]**	Accept answers between 53 and 54mm.
6	(c)		Given: $\sin 30^0 + \cos 60^0 + \tan 45^0 = 2$ LHS = 0.5 + 0.5 + 1 = 2 RHS = 2 Therefore 2 = 2 QED	**[1]** **[1]**	
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
7	(a)		Integrate $\cos 2x + 1/x^2 + \sqrt{x}$ with respect to x. So $\int \cos 2x + 1/x^2 + \sqrt{x}\ dx = \int \cos 2x + x^{-2} + x^{0.5}\ dx$ $= \frac{1}{2} \sin 2x + (x^{-1})/-1 + (x^{1.5})/1.5 + C$ $= \frac{1}{2} \sin 2x - 1/x + \frac{2}{3}\sqrt{x^3} + C$	**[5]**	Award one mark for $\int \cos 2x + x^{-2} + x^{0.5}\ dx$. Award one mark for $\frac{1}{2} \sin 2x$ Award one mark for $-1/x$ or $-x^{-1}$ Award one mark for $+\frac{2}{3}$ or 1/1.5 Award one mark for $\sqrt{x^3}$ or $x^{3/2}$ or $x^{1.5}$ +C is not required for full marks
7	(b)		Given $v = 4 + 6t$ where $t_1 = 0$ and $t_2 = 5$. Distance $= \int_0^5 (4 + 6t)\ dt$ $= [4t + 3t^2]_0^5$ $= (\{4 \times 5\} + \{3 \times 5^2\}) - 0$ $= 95$ metres	**[1]** **[1]** **[1]**	Award one mark for Distance $= \int_0^5 (4 + 6t)\ dt$. Award one mark for $[4t + 3t^2]_0^5$ Unit not required for full marks

Question			Expected Answer	Mark	Rationale/Additional Guidance
7	(c)		Given $F = 8s - s^2$ where $s_1 = 2$ and $s_2 = 6$ Work done $= \int_2^6 (8s - s^2)\, ds$ $= [4s^2 - \frac{1}{3}s^3]_2^6$	[1]	
			$= (\{4 \times 6^2\} - \{\frac{1}{3} \times 6^3\}) - (\{4 \times 2^2\} - \{\frac{1}{3} \times 2^3\})$ $= (144 - 72) - (16 - 2\frac{2}{3})$ $= 72 - 13\frac{1}{3}$ $= 58\frac{2}{3}$ or 58.666… or 58.67 newtons	[1]	
			Total	[10]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
8	(a)		An event that is affected by previous events. Suitable example involving an experiment 'without replacement' or equivalent.	[1] [1]	
8	(b)		Total number of screws = 110 + 120 + 70 = 300. Assume that three separate draws take place. Draw One: Probability of selecting a steel screw is 120/300. Draw Two: There are now 119 steel screws in a total of 299 Probability of selecting a steel screw is 119/299. Draw Three: There are now 118 steel screws in a total of 298. Probability of selecting a steel screw is 118/298 The probability of choosing a steel screw on the first draw and the second and the third draw is: (120/300) x (119/299) x (118 x 298) is 0.063	[1] [1] [1] [1]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
8	(c)		Mutually exclusive Not Mutually exclusive Not Mutually exclusive Mutually exclusive	[4]	Award one mark for each correct response.
			Total	[1]	

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