OCR
Oxford Cambridge and RSA

Level 3 Certificate

Mathematical Techniques and Applications for Engineers

Unit **H865/01** Component 1

OCR Level 3 Certificate

Mark Schemes for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

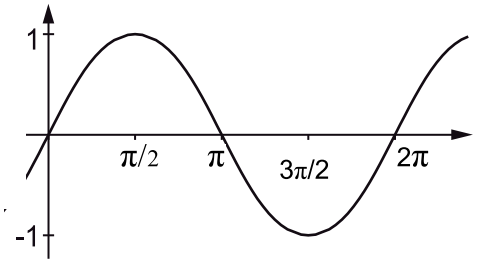
OCR will not enter into any discussion or correspondence in connection with this mark scheme.

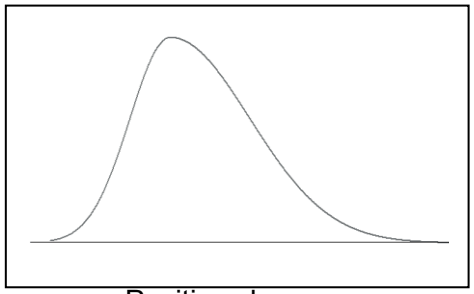
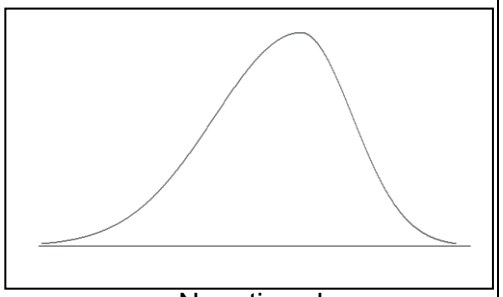
These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Stamp	*Description*
	Tick
	Cross
	Error carried forward
	Benefit of doubt

General :
Award full marks for correct answer seen without working unless otherwise stated.

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section A					
1			$4(2x + 5) – 3x = 8x + 20 – 3x = 5x + 20$ $5x$ 20	[1] [1]	
2			$x^2 + 9x + 20 = (x + 4)(x + 5)$ $(x + 4)$ $(x + 5)$	[1] [1]	
3			$(x + 3)/2 – (x + 4)/5 = (5x + 15 – 2x – 8)/10 = (3x + 7)/10$ $(3x + 7)$ 10	[1] [1]	Award two marks for $0.3x + 0.7$.
4			$5(x + 2) - 6(x – 3) = 0$ $5x + 10 – 6x + 18 = 0$ $– x + 28 = 0$ $x = 28$	[1] [1]	Award one mark for $5x + 10 – 6x + 18 = 0$ or $– x + 28 = 0$. Award one mark for $x = 28$.
5			One revolution = 2π radians Speed = $50 \times 2\pi = 100\pi = 314.2$ radians per minute	[1] [1]	Award one mark for one revolution = 2π radians. Award one mark for 100π or answers between 314 and 315 with or without units.
6			Given $i = I \sin ø$		Award one mark for 30^o and award one mark

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
	When i = 5 A and I = 10 A then 5 = 10 sin ø So ø = $\sin^{-1}$ 5/10 = 30° or 150°	**[2]**	for 150°.
7	Given sec x. – sec x. $\sin^2$ x So sec x(1 - $\sin^2$ x) But 1- $\sin^2$ x = $\cos^2$ x and sec x = 1/cos x So sec x(1 - $\sin^2$ x) = $\cos^2$ x/cos x = cos x	**[2]**	Award one mark for sec x(1 - $\sin^2$ x) or 1- $\sin^2$ x = $\cos^2$ x and sec x = 1/cos x or sec x(1 - $\sin^2$ x) = $\cos^2$ x/cos x. Award one mark for cos x.
8		**[2]**	Award one mark for correct positive half cycle and award one mark for correct negative half cycle.
9	Given y = 1/x = x^{-1} So dy/dx = – x^{-2} = – 1/x^2 x^{-1} – x^{-2} or – 1/x^2	**[1]** **[1]**	
10	Given y = 3 sin 4x Then dy/dx =(3 x 4) cos 4x = 12 cos 4x 12 Cos 4x	**[1]** **[1]**	

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
11	$\int \sqrt{x}\ dx = \int x^{½}\ dx$ $= (x^{1.5})/1.5 + C$ $= ⅔\ \sqrt{x^3} + C$ $⅔\ \sqrt{x^3}$ C	**[1]** **[1]**	Also accept $\frac{2x^{\frac{3}{2}}}{3} + C$ OE
12	In bar charts, the groups are typically categorical variables i.e. the width of each bar is the same In histograms the groups are typically intervals of another continuous variable i.e. the width of each bar is different.	**[1]** **[1]**	Also award marks for In a bar chart the **frequency** is indicated by the **height** of the bar i.e. the y axis indicates the frequency. In a histogram the frequency is indicated by the **area** of the bar i.e. the y axis indicates the **frequency density**. Also in a histogram the bars CAN HAVE different widths and always **touch** at their boundaries. In a bar chart the width of the bars are not significant and normally **do not touch** each other.
13	.  Positive skew  Negative skew	**[2]**	Award one mark for each correct skew.

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section A					
14			Probability = 1 – (3/5 + 1/4) = 3/20	**[1]** **[1]**	
15			Assume the 12 components are place in a random order. P(last component is white) = (number of whites)/(total number of components) Probability = 7/12	**[2]**	Award one mark for understanding. Award one mark for 7/12.
			Total	**[30]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
Section B					
1	**(a)**		Given $PV = mRT$ Divide both sides by mR Then $T = PV/mr$	**[1]**	
1	**(b)**		Given $L_t = L_o(1 + \alpha t)$ Open brackets $L_t = L_o + L_o \alpha t$ Then $L_o \alpha t = L_t - L_o$ So $\alpha = (L_t - L_o)/ L_o t$	**[1]** **[1]** **[1]**	Accept any other correct method.
1	**(c)**		Given $D/d = \sqrt{[(f + p)/(f - p)]}$ Square both sides Then $D^2/d^2 = (f + p)/(f - p)$ Cross multiply Then $D^2(f - p) = d^2(f + p)$ Open brackets Then $D^2f - D^2 p = d^2f + d^2 p$ So $D^2f - d^2f = d^2 p + D^2 p$ Then $f(D^2 - d^2) = p(d^2 + {}^2 p)$ So $f = p(d^2 + D^2)/(D^2 - d^2)$	**[1]** **[1]** **[1]** **[1]** **[1]** **[1]**	Accept any other correct method.
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
2	(a)		Given $x^2 - 10x + 24 = 0$ By inspection $x^2 = (x, x)$ 24 = (1, 24) or (2, 12) or (3, 8) and (4, 6) By trial and error $(x - 4)(x - 6) = 0$ So $(x - 4) = 0$ and $(x - 6) = 0$ Therefore x = 4 or 6	[3]	Award one mark for $(x - 4)(x - 6) = 0$. Award one mark for $(x - 4) = 0$ and $(x - 6) = 0$. Award one mark for x = 4 or 6.
2	(b)	(i)	Original area of sheet metal = 8 x 5 = 40 m^2 New area of sheet metal = 40 + 50% of 40 = 60 m^2 Original length = 8 m New length = x + 8 Original width = 5 m New width = x + 5 Therefore in terms of x New area = (x + 8)(x + 5) So (x + 8)(x + 5) = 60 Then $x^2 + 5x + 8x + 40 = 60$ So $x^2 + 13x - 20 = 0$ QED	[4]	Award one mark for 60 m^2. Award one mark for new length = x + 8 and new width = x + 5. Award one mark for (x + 8)(x + 5) = 60. Award one mark for $x^2 + 5x + 8x + 40 = 60$ or $x^2 + 13x - 20 = 0$.
2	(b)	(ii)	Given $x^2 + 13x - 20 = 0$ Use the quadratic formula $x = [- b \pm \sqrt{(b^2 - 4ac)}]/(2a)$ Where a = 1, b = 13 and c = – 20. So $x = [- 13 \pm \sqrt{(13^2 - 4x1 \times -20)}]/2$ $x = [- 13 \pm \sqrt{[(169 + 80)]}/2$ $x = [- 13 \pm \sqrt{[(169 + 80)]}/2$ $x = [- 13 \pm \sqrt{[(249]}/2$ $x = [- 13 \pm 15.78]/2$ x = 1.39 m	[3]	Award one mark for $x = [- b \pm \sqrt{(b^2 - 4ac)}]/(2a)$ Award one mark for intermediate calculations. Award one mark for answers between 1.35 and 1.45 with or without units.
			Total	[10]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
3	(a)	(i)	cos BCA = 1.5/4.2 Angle BCA = $\cos^{-1}$ 1.5/4.2 = 69.1° Assumes base of ladder is 1.5 m from base of wall.	[2]	Award one mark for cos BCA = 1.5/4.2. Award one mark for angle BCA between 69 and 69.5° with or without units.
		(ii)	The ladder is safe because angle BCA is 69.1° which comes inside the safety limits of 68 to 82°.	[1]	**Accept ECF from part(i)** **So unsafe if answer in (i) is outside safety limits.**
3	(b)	(i)	AB = 20 cos 30° = 17.32 m BD = 20 sin 30° = 10 m	[4]	Award one mark for AB = 20 cos 30°. Award one mark for 17.32 m with or without the unit. Award one mark for BD = 20 sin 30°. Award one mark for 10 m with or without the unit.
		(ii)	Angle C = $\sin^{-1}$ 10/15 = 41.8°	[1]	
		(iii)	BC = 15 cos 41.8° = 11.18 m or BC = $\sqrt{(15^2 + 10^2)}$ = 11.8 m	[1]	
		(iv)	AB = 11.18 + 17.32 = 28.5 m	[1]	
			Total	[10]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
4	(a)		Let angle of elevation be x^o then $\tan x^o = 20/40 = 0.5$ so angle $x^0 = \tan^{-1} 0.5 = 26.57^o$ Assumes that: 'A **vertical** post of height 1 m and a **vertical** radio mast of 21 m are mounted on **horizontal ground** with a distance of 40 m **between their bases.** Calculate the angle of elevation to the top of the radio mast **from** the top of the post**.'**	**[2]**	Award one mark for $\tan x = 20/40 = 0.5$. Award one mark for angle $x = \tan^{-1} 0.5$ or answers between 26 and 27^o with or without units.
4	(b)	(i)	Angle A + Angle B + Angle C = 180^o So Angle B = 180^o – Angle A – Angle B = $180^o – 80^o – 45^o$ = 55^o	**[2]**	Award one mark for Angle A + Angle B + Angle C = 180^o or Angle B = 180^o – Angle A – Angle B. Award one mark for 55^o.
4	(b)	(ii)	$AB/\sin 45^o = 5/\sin 55^o$ $AB = (5 \sin 45^o)/\sin 55^o$ = 4.316 m	**[3]**	Award one mark for $AB/\sin 45^o = 5/\sin 55^o$. Award one mark for $AB = (5 \sin 45^o)/\sin 55^o$. Award one mark for answers between 4.3 and 4.4 m with or without units.
4	(b)	(iii)	Area of triangle = ½ (AC)(AB) sin A^o = ½ x 5 x 4.316 sin 80^o = 10.626 m^2	**[3]**	Award one mark for Area = ½ (AC)(AB) sin A^o. Award one mark for ½ x 5 x 4.316 sin 80^o. Award one mark for answers between 10 and 11 m^2 with or without units.
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
5	(a)		Given $y = 2x^3 – 24x – 1$ Then $dy/dx = 6x^2 – 24$ At zero gradient $dy/dx = 0$ Therefore $6x^2 – 24 = 0$ So $6x^2 = 24$ and $x = \pm 2$ Assumes that values of x are required	[2]	Award one mark for $dy/dx = 6x^2 – 24$. Award one mark for $x = \pm 2$.
5	(b)		$d^2y/dx^2 = 12x$ At $x = +2$ then $d^2y/dx^2 = 12 \times 2 = +24$ When d^2y/dx^2 gives a positive value we have a minimum value for y. So $y_{min} = 2x^3 – 24x – 1$ $= (2 \times 2^3) – (24 \times 2) – 1$ $= 16 – 48 – 1$ $= – 33$ Alternative solution y at $x = 2$ $= (2 \times 2^3) – (24 \times 2) – 1$ $= 16 – 48 – 1$ $= – 33$	[4]	Award one mark for $d^2y/dx^2 = 12x$. Award one mark for $x = +2$ then $d^2y/dx^2 = +24$. Award one mark for $y_{min} = 2x^3 – 24x – 1$ or $(2 \times 2^3) – (24 \times 2) – 1$ or $16 – 48 – 1$. Award one mark for – 33.
5	(c)		$d^2y/dx^2 = 12x$ At $x = – 2$ then $d^2y/dx^2 = 12 \times –2 = –24$ When d^2y/dx^2 gives a negative value we have a maximum value for y. So $y_{max} = 2x^3 – 24x – 1$ $= (2 \times –2^3) – (24 \times – 2) – 1$ $= –16 + 48 – 1$ $= + 31$	[4]	Award one mark fo $x = – 2$ then $d^2y/dx^2 = –24$. Award one mark for d^2y/dx^2 gives a negative value we have a maximum value for y. Award one mark for $y_{max} = 2x^3 – 24x – 1$ or $(2 \times –2^3) – (24 \times – 2) – 1$ or $–16 + 48 – 1$ Award one mark for + 31.

Question			Expected Answer	Mark	Rationale/Additional Guidance
			Alternative solution y at x = -2 $= 2x^3 – 24x – 1$ $= (2 \times –2^3) – (24 \times – 2) – 1$ $= –16 + 48 – 1$ $= + 31$ So Maximum at (-2, 31) Minimum at (2, -33)		
			Total	**[10]**	

Question			Expected Answer	Mark	Rationale/Additional Guidance
6	(a)	(i)	Given $x = 10t^3 + 6t^2 – 4t + 3$ Velocity = $dx/dt = 30t^2 + 12t – 4$ When t = 2 seconds Velocity = $(30 \times 2^2) + (12 \times 2) – 4$ $= 140$ m s^{-1}	**[2]**	Award one mark for Velocity = $dx/dt = 30t^2 + 12t – 4$. Award one mark for Velocity = $(30 \times 2^2) + (12 \times 2) – 4$ or 140 m s^{-1} with or without units.
6	(a)	(ii)	$dx/dt = 30t^2 + 12t – 4$ Acceleration = $d^2x/dt^2 = 60t + 12$ When t = 2 seconds Acceleration = $(60 \times 2) +12$ $= 132$ m s^{-2}	**[2]**	Award one mark for Acceleration = $d^2x/dt^2 = 60t + 12$. Award one mark for Acceleration = $(60 \times 2) +12$ or 132 m s^{-2} with or without units.
6	(b)		Given $dy/dx = 4x – (10/\sqrt{x}) – 5$ Then $y = \int (4x – (10/\sqrt{x}) – 5)\, dx$ $= \int (4x – (10\, x^{-½}) – 5)\, dx$ $= 2x^2 – (20\sqrt{x}) – 5x + C$ At point P, x = 4 and y = 6 So $6 = (2 \times 4^2) – (20\ \sqrt{4}) – (5 \times 4) + C$ $6 = 32 – 40 – 20 + C$ So C = 34 Therefore $y = f(x) = 2x^2 – (20\sqrt{x}) – 5x + 34$	**[6]**	Award one mark for $y = \int (4x – (10/\sqrt{x}) – 5)\, dx$. Award one mark for $y = \int (4x – (10\, x^{-½}) – 5)\, dx$. Award one mark for $y = 2x^2 – (20\sqrt{x}) – 5x + C$. Award one mark for $6 = (2 \times 4^2) – (20\ \sqrt{4}) – (5 \times 4) + C$ or $6 = 32 – 40 – 20 + C$. Award one mark for C = 34. Award one mark for $y = f(x) = 2x^2 – (20\sqrt{x}) – 5x + 34$.
			Total	**[10]**	

Question	Expected Answer	Mark	Rationale/Additional Guidance
7 (a)		[6]	

X	$X - \bar{X}$	$(X - \bar{X})^2$
5	– 4	16
7	– 2	4
9	0	0
11	2	4
13	4	16
$\sum X = 45$		$\sum(X - \bar{X})^2 = 40$

Mean $\bar{X}$ = 45/5 = 9

Standard deviation s = √ (40/5) = 2.828

Award one mark for $\sum X = 45$.

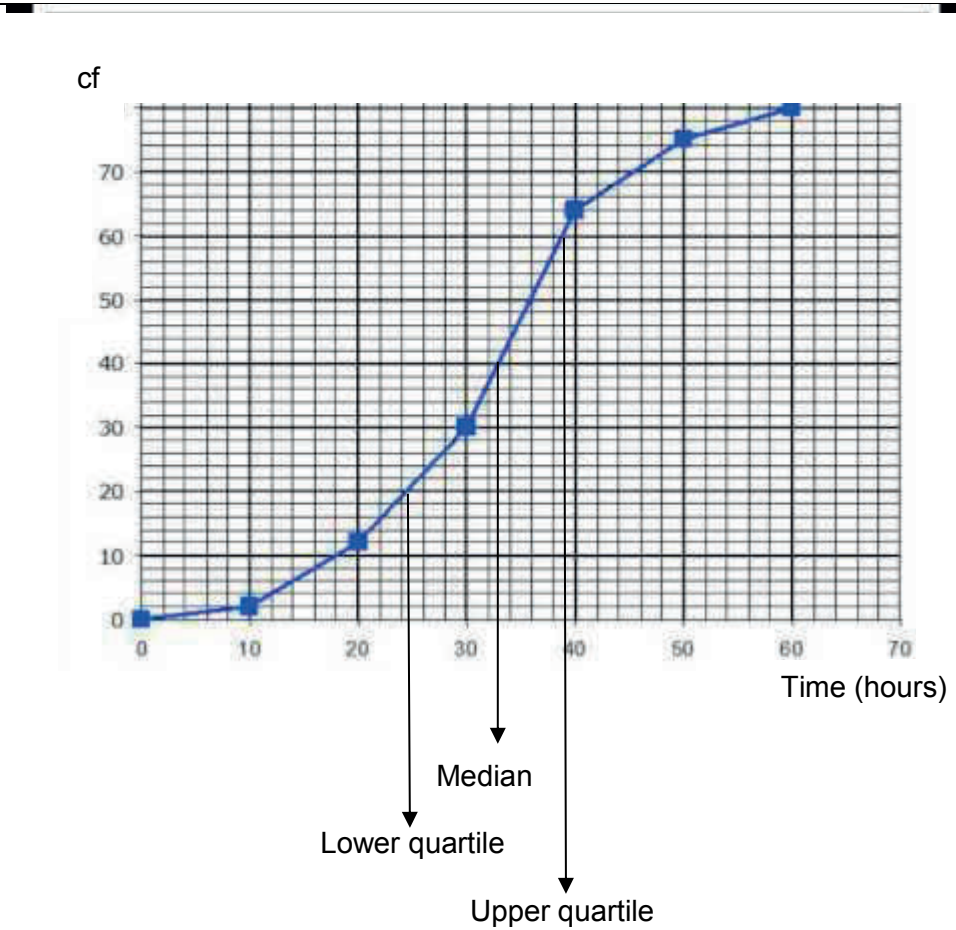
Award one mark for Mean $\bar{X}$ = 45/5 = 9.

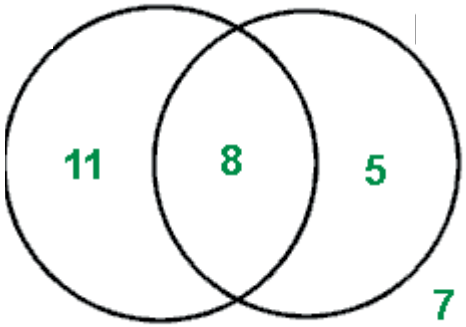
Award one mark for $X - \bar{X}$ column.

Award one mark for $(X - \bar{X})^2$ column.

Award one mark for $\sum(X - \bar{X})^2 = 40$.

Award one mark for s = √ (40/5) = 2.828

7	(b)			[4]	
		(i)	Median mark = 33		Award one mark for a median mark of between 32 and 34.
		(ii)	Lower quartile = 24.5		Award one mark for a lower quartile mark of between 24 and 26.
		(iii)	Upper quartile = 39		Award one mark for an upper quartile mark of between 38 and 40.
		(iv)	60 hours		Award one mark for 60 hours.
			Total	**[10]**	

<table>
<tr><th colspan="3">Question</th><th>Expected Answer</th><th>Mark</th><th>Rationale/Additional Guidance</th></tr>
<tr><td>8</td><td>(a)</td><td></td><td>
<table>
<tr><th>Statement</th><th></th></tr>
<tr><td>If P(A) equals zero, event A will almost definitely not occur.</td><td>✓</td></tr>
<tr><td>If P(A) is close to zero, there is a small chance that event A will occur.</td><td>✓</td></tr>
<tr><td>If P(A) equals zero, event A will almost definitely occur.</td><td></td></tr>
<tr><td>If P(A) equals 0.5, there is a 50-50 chance that event A will occur.</td><td>✓</td></tr>
<tr><td>If P(A) equals 0.5, there is a 50-50 chance that event A will not occur.</td><td>✓</td></tr>
<tr><td>If P(A) is close to one, there is a strong chance that event A will occur.</td><td>✓</td></tr>
<tr><td>If P(A) equals one, event A will almost definitely occur.</td><td>✓</td></tr>
<tr><td>If P(A) equals one, event A will almost definitely not occur.</td><td></td></tr>
</table>
</td><td>[5]</td><td>If there are more than five ✓ award zero.
If there are up to five ✓ award one mark for each correct response.</td></tr>
<tr><td>8</td><td>(b)</td><td>(i)</td><td> 
</td><td>[4]</td><td>There are 8 learners who can operate a centre lathe and a milling machine go in the intersection because they need to be in both circles.
The 7 learners who cannot operate a centre lathe or milling machine go on the outside.
There are 13 learners who operate a milling machine, so the numbers in the milling machine circle should add up to 13.
We already have 8 in the intersection, so there must be 5 who operate a milling machine but not a centre</td></tr>
</table>

			As the question is written the 11 should be19 and the 5 should 13. I have suggested the word 'only'' be removed from the question and if so the answer given is correct.		lathe. There are 19 learners who can only operate a centre lathe so the number who can operate a centre lathe but not a milling machine must be 19 – 8 = 11. Award one mark for each correct number.
8	**(b)**	**(ii)**	Number in class = 11 + 8 + 5 + 7 = 31 47 if 'only' is not removed.	**[1]**	
			Total	**[10]**	

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