OCR
RECOGNISING ACHIEVEMENT

LEVEL 3 CERTIFICATE
MATHEMATICS FOR ENGINEERING
Paper 1

H860/01

Candidates answer on the answer booklet.

OCR supplied materials:
- 16 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 2 hours

* H 8 6 0 0 1 *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 The impedance, Z ohms (Ω), of an electrical circuit consisting of a resistor and coil connected in series is

$$Z = R + j\omega L,$$

where $j = \sqrt{-1}$,
R is the value of the resistance in ohms (Ω),
ω is the frequency of an applied voltage in radians per second (rad s^{-1}),
L is the value of the inductance of the coil in henry (H).

(a) The resistance is $400\,\Omega$, the inductance is 0.1 H and the frequency is $5000\,\text{rad s}^{-1}$.

(i) Draw an Argand diagram to represent the impedance Z. **[1]**

(ii) Express the impedance, Z, in polar form. **[3]**

(iii) The current, I amperes, passing through the circuit is

$$I = \frac{v}{Z},$$

where v volts is the applied root mean square (r.m.s.) voltage.

Calculate the current passing through the circuit when $v = 10$. Give your answer in the form $a + jb$. **[2]**

(b) The impedance, Z, of a particular LCR circuit is given by

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C.$$

Show that $\frac{1}{Z}$ may be written as

$$\frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right).$$ **[2]**

2 The coefficient of linear expansion, α, of a solid is the amount by which a unit length of the material expands when the temperature increases by one degree Celsius.

In the case of a rod,

$$l_t = l_0(1 + \alpha t),$$

where l_0 is the initial length,
l_t is the final length,
t is the rise in temperature in °C.

(a) An aluminium rod has length 300 mm when its temperature is 20 °C. Calculate the length of the rod when its temperature increases to 120 °C. The coefficient of linear expansion of aluminium is 26×10^{-6} per °C. **[2]**

(b) A square metal plate has sides of length b mm at 20 °C. It is heated to 120 °C. Assume that the plate expands uniformly in each direction.

(i) Express the area of the plate after expansion in terms of the original length b and the coefficient of linear expansion α. **[2]**

When heated, the area of the plate increases by 0.5%.

(ii) Calculate the coefficient of linear expansion for this metal. **[3]**

3 Solar panels are often mounted on horizontal roofs. Fig. 3 shows panels tilted at an angle θ. In order to maximize the number of panels, the distance, d, between each panel should be small. However, there is no advantage in allowing d to be very small since each panel will partly obscure the Sun's radiation from the panel behind. Fig. 3 shows panels of length l separated in such a way that no part of any panel is shaded by the panel in front when the Sun has an angle of elevation α. If the Sun is lower in the sky, then each panel will be in partial or total shade.

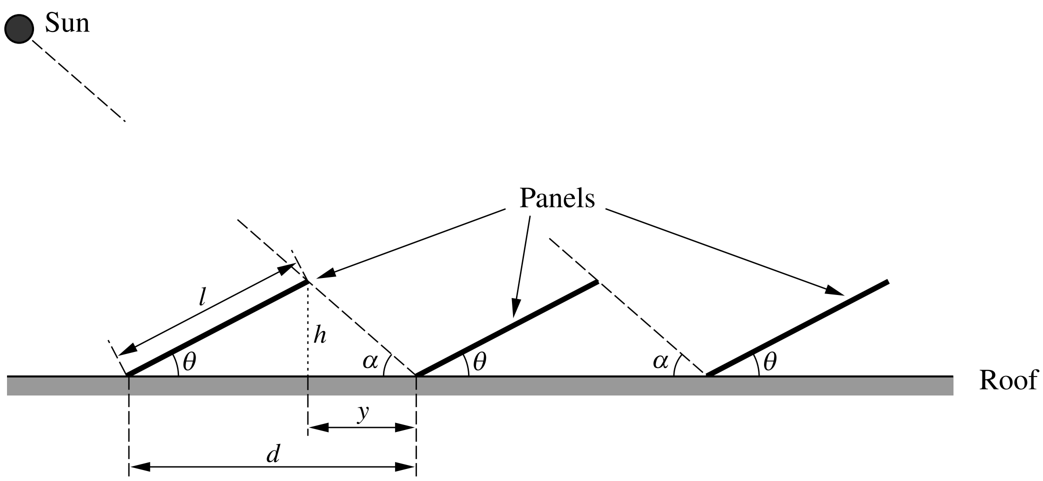

Fig. 3

(a) Express the distance y indicated in Fig. 3 in terms of d, l and θ. **[1]**

(b) Express the height, h, of the panel indicated in Fig. 3 in terms of

(i) l and θ,

(ii) y and α. **[2]**

(c) The utilisation, u, of the roof space is defined as

$$u = \frac{l}{d}.$$

Using your results in parts **(a)** and **(b)**, show that

$$\alpha = \tan^{-1}\left(\frac{u\sin\theta}{1 - u\cos\theta}\right).$$ **[3]**

4 **(a)** Find the following integrals.

(i) $\int e^{\frac{1}{4}x}\,dx$ **[1]**

(ii) $\int xe^{\frac{1}{4}x}\,dx$ **[2]**

(iii) $\int x^2 e^{\frac{1}{4}x}\,dx$ **[4]**

(b) The volume, V, generated when a plane figure bounded by the curve with equation $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$, rotates completely about the y-axis is given by

$$V = 2\pi \int_a^b xy\,dx.$$

A metal component shown in Fig. 4 is to be turned on a lathe. The top curved surface is generated by the curve with equation

$$y = xe^{\frac{1}{4}x} + 1 \quad \text{for} \quad 1 \leqslant x \leqslant 2.$$

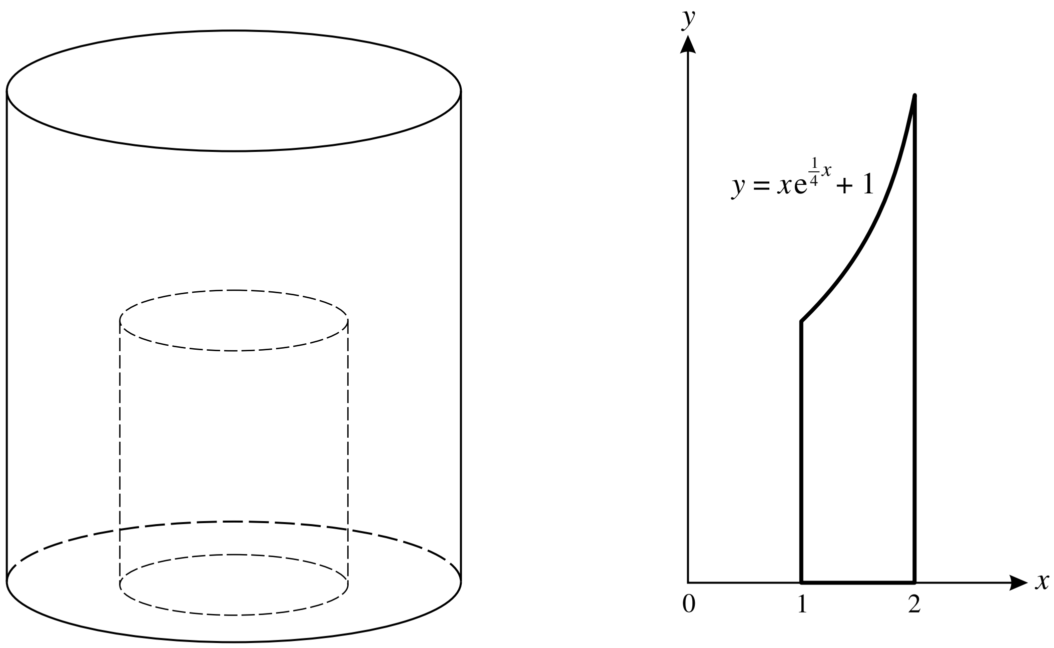

Fig. 4

Calculate the volume of material in the finished component. **[4]**

5 The outline of a simple crank and piston mechanism is shown in Fig. 5. The length of the crank, OC, is r mm and the length of the connecting rod, CP, is l mm. The piston slides backwards and forwards along the horizontal line OP while the crank rotates about the fixed point O with angular velocity ω rad s^{-1}.

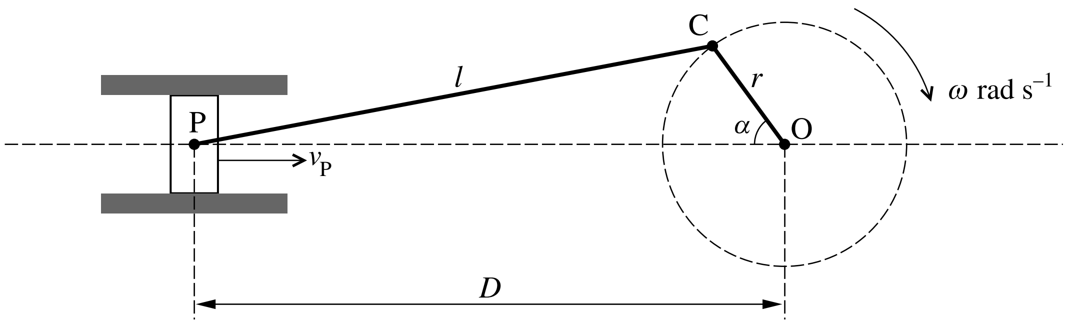

Fig. 5

(a) Show that when the crank makes an angle α with OP, the distance, D, between O and P is given by

$$D = r\cos\alpha + l\sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2\alpha}.$$ **[3]**

(b) By using a binomial expansion, or otherwise, show that when r is sufficiently smaller than l

$$\sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2\alpha} \approx 1 - \frac{r^2}{2l^2}\sin^2\alpha.$$ **[3]**

(c) By using this approximation in the formula for D given in part **(a)** and then differentiating, show that the velocity of the piston at time t, $v_P(t)$, can be approximated by

$$v_P(t) \approx -r\omega\left(\sin\omega t + \frac{r}{l}\sin\omega t\cos\omega t\right).$$ **[3]**

6 A rectangular water tank is 800 mm long, 500 mm wide and 1000 mm high. The tank is fed by a supply of 4 litres per second. When the tank is being drained, water escapes through a valve at a rate that is directly proportional to the height, h mm, of the water in the tank. In this case the water escape rate is kh litres per second, where k is a constant.

(a) Starting with the tank empty, the water supply is turned on and the drain valve remains closed so that no water escapes.

Derive a formula that relates the height of the water in the tank to the time in seconds for which the water supply is on. **[2]**

(b) When the height of the water reaches 800 mm the supply is turned off. The drain valve is then opened and water begins to escape.

Derive a formula that relates the height of the water in the tank to the constant k and to the time in seconds for which the drain valve is open. **[4]**

(c) When the height of the water falls to 400 mm, the supply is turned on again; the drain valve remains open.

Derive a formula that relates the height of water in the tank to the constant k and to the time in seconds for which the water supply is on and the drain valve is open. **[5]**

7 In computer networks, information is transmitted in packets containing digital values. Sometimes the information in a packet becomes corrupted due to electrical interference. When this happens the packet is transmitted again in the hope that there will be no corruption on the second occasion. If there is further corruption the packet is transmitted again and this process is continued until the packet has been transmitted successfully.

The probability that an individual packet becomes corrupted during a transmission is p, which is independent of the success or failure of any preceding transmissions.

(a) If $p = 0.01$, what is the probability that a packet will have to be transmitted exactly three times? **[2]**

(b) Show that the average number of transmissions required in order for a packet to be transmitted successfully is given by

$$\sum_{r=1}^{\infty} r\left(p^{r-1} - p^{r}\right).$$ **[3]**

(c) Show that the average number of transmissions required in order for a packet to be transmitted successfully may also be expressed as

$$\frac{1}{1-p}.$$ **[3]**

