LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

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Paper 2

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Wind Power

In July 2009 the British Government published its Renewable Energy Strategy, which plans to reduce carbon emissions by 34% over the following decade. Among the proposals for implementation is the rapid deployment of wind turbines, which is likely to include 4000 new onshore installations and 3000 new installations at sea. It is clear that wind power will play a significant role in the reduction of carbon emissions in the future; however, wind turbines capture less than 50% of the power contained in the wind in an area swept by the turbine blades.

Large wind turbines typically have three rotor blades of over 20 m in length mounted on a central shaft. The blades are designed in such a way that the wind causes them to turn, rotating the shaft and driving an attached electrical generator, normally via a gear box. In optimum conditions large wind turbines can generate several megawatts of power. Conventional wind turbines, such as the one shown in Fig. 1, are mounted on a tower and continually adjust their position to face directly into the wind.

The power, P_W watts, of the wind passing through a given area, A m^2, is

$$P_W = \tfrac{1}{2}\rho A v_1^3$$

where ρ kg m^{-3} is the density of air and v_1 m s^{-1} is the speed of the wind.

The density of air reduces with increasing altitude and changes with variations in temperature. At sea level and 20 °C, air has a density of approximately 1.2 kg m^{-3}.

Not all the power of the wind can be converted into electrical power by a wind turbine. The electrical power, P_E watts, captured by the turbine is

$$P_E = C_P P_W = \tfrac{1}{2} C_P \rho A v_1^3$$

where C_P is called the power coefficient and is the proportion of the wind power that can be converted into electrical power, and A is the area swept by the rotor.

A large turbine with a sweep diameter of 44 m (blades of length 22 m), an approaching wind speed of 7 m s^{-1}, a power coefficient of 0.4 and density of air of 1.2 kg m^{-3} gives a value of about 125 kW for P_E.

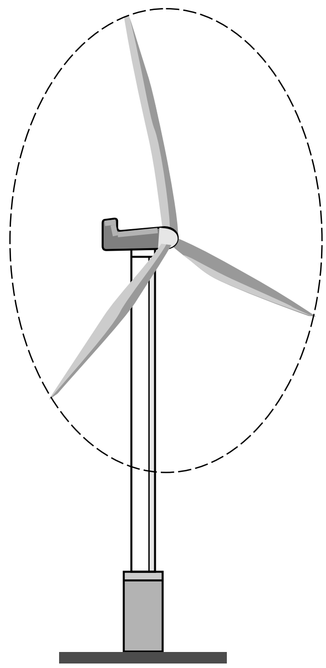

Fig. 1

The value of the power coefficient is a result of several factors. Albert Betz developed a theory which determines the maximum power that can be extracted by a wind turbine. The maximum value of the power coefficient according to this theory is called the *Betz limit*.

Wind speed is reduced as it passes through the swept area of the rotor. If the wind speed is v_1 as it approaches the swept area and v_2 as it leaves the swept area, then the power, P_T, taken from the wind is given by

$$P_T = \tfrac{1}{4}\rho A (v_1 + v_2)(v_1^2 - v_2^2).$$

The electrical power P_E produced by the turbine will be less than this as a result of other factors such as rotor blade drag, mechanical friction, copper losses and rectifier losses. Depending upon the design of the turbine and wind speeds, power coefficients of between 0.2 and 0.4 are typical.

Turbines are designed to provide optimum electrical power for particular ranges of wind speeds. Manufacturers will often state the total power output of turbines at an optimum wind speed. However, wind speeds change dramatically with time and it is useful to determine the distribution of wind speeds at particular sites. This is done by taking detailed measurements and tabulating the proportion of time that each range of speeds occurs. Experimentation shows that a good approximation to variation in wind speeds is given by the probability density function f(v), given by

$$f(v) = \frac{\pi}{2}\frac{v}{\bar{v}^2}e^{-\frac{\pi}{4}\frac{v^2}{\bar{v}^2}}$$

where v is the speed and $\bar{v}$ is the mean speed. Fig. 2 shows this probability density function for two different values of $\bar{v}$.

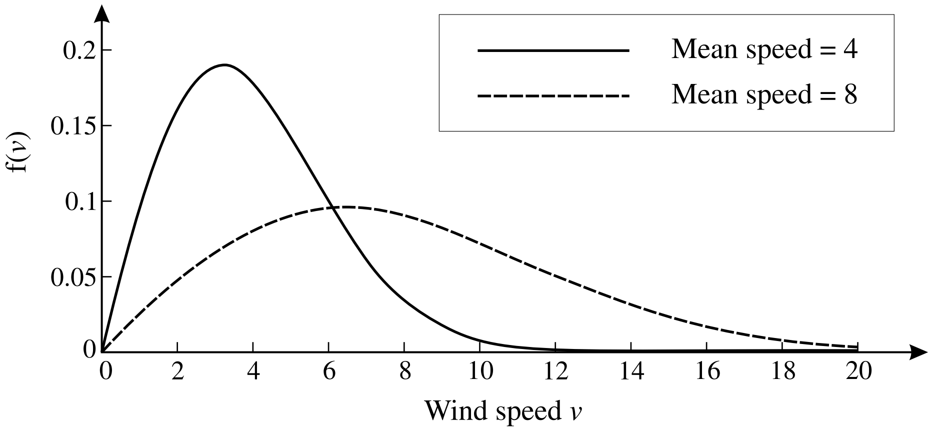

Fig. 2

This probability density function can then be used to determine the probability of finding wind speeds in a particular interval. For obvious reasons, wind turbines should be sited where average wind speeds are fairly high. However, very high wind speeds will take the wind turbine past its safe working limit and lead to damage of both mechanical and electrical components. In these situations a brake is applied to limit the angular speed of the rotor.

Although wind speed is a significant factor in the electrical power that a turbine can produce, it is the *rotor tip speed ratio* that provides a more realistic measure to determine the power coefficient. The rotor tip speed ratio is the ratio of the speed of the tip of the rotor to the speed of the wind; typical values range from 4 to 10. The graph that relates the dependence of power coefficient to tip speed ratio is usually plotted from measurements taken at particular sites and has the general shape shown in Fig. 3. The formula for this relationship can be approximated by placing a curve through a number of known measurements.

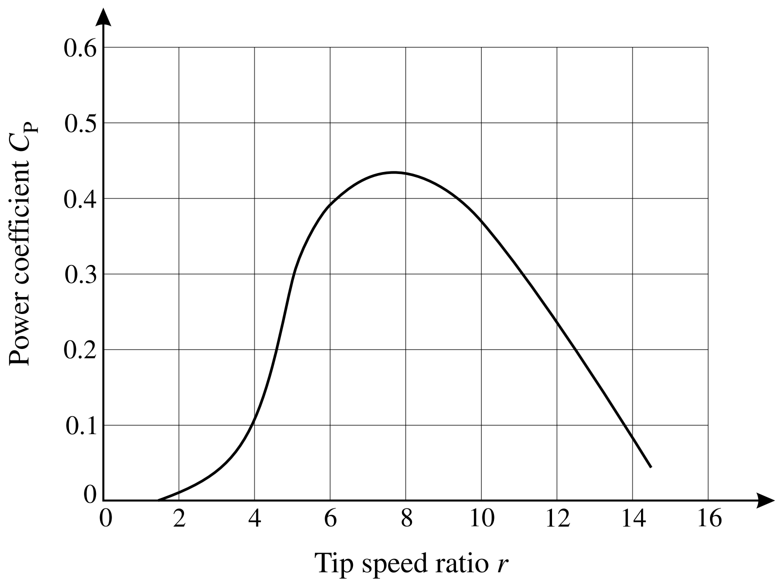

Fig. 3

Current developments taking place in the design of wind turbines and the careful siting of wind turbine farms seek to optimise the amount of electrical power that can be produced; however, the Betz limit is the main factor that places a maximum value of about 0.6 on the power coefficient of any wind turbine. It remains to be seen how effective wind power will become at providing a significant input to the National Grid.