

Friday 13 January 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/01 Paper 1

Candidates answer on the Answer Booklet.

OCR supplied materials:

- 8 page Answer Booklet (sent with general stationery)
- Graph paper
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 2 hours



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 (a) A manufacturer mass produces filament lamps and experience shows that the standard deviation of the lifetimes of the lamps remains constant.
Last year the manufacturer conducted a test on a random sample of 2000 lamps. The times, t hours, that the lamps lasted before they failed are summarised in Table 1.

Time in hours	Number failing
$t \leq 200$	0
$200 < t \leq 400$	50
$400 < t \leq 600$	300
$600 < t \leq 800$	420
$800 < t \leq 1000$	500
$1000 < t \leq 1200$	400
$1200 < t \leq 1400$	250
$1400 < t \leq 1600$	50
$1600 < t \leq 1800$	30
$1800 < t$	0

Table 1

- (i) Draw a histogram to represent the results shown in the table. [2]
- (ii) Estimate the mean and standard deviation of the lifetime of the lamps in this sample. Give your answers correct to two significant figures. [5]
- (b) The quality control department now suspects that the mean lifetime of the lamps has increased since the test last year. They select a random sample of 50 lamps and find that the average lifetime is 975 hours.

Based on these results, test the hypothesis, at the 1% significance level, that lamps produced now have an increased mean lifetime.

You may assume that the mean and standard deviation calculated in part (a) are the values for the entire population. [4]

- 2 (a) A right-angled triangle, HAG, has fixed hypotenuse $HG = h$. Prove that the area of HAG is maximised when the lengths of the two perpendicular sides, AH and AG, are equal so that

$$AH = AG = \frac{h}{\sqrt{2}}. \quad [4]$$

- (b) A diagram of a roof-supporting structure containing straight, rigid members is shown in Fig. 2. The lengths of members HE, GE and HG are 3 m, 2 m and 4 m respectively. The structure is symmetrical about the line AH, and such that the area of the triangle FHG is maximised.

- (i) Determine the height of the structure, AH. [1]
 (ii) Calculate the angles $\widehat{GHE}$ and $\widehat{HGE}$. [3]
 (iii) Calculate the length BC. [2]

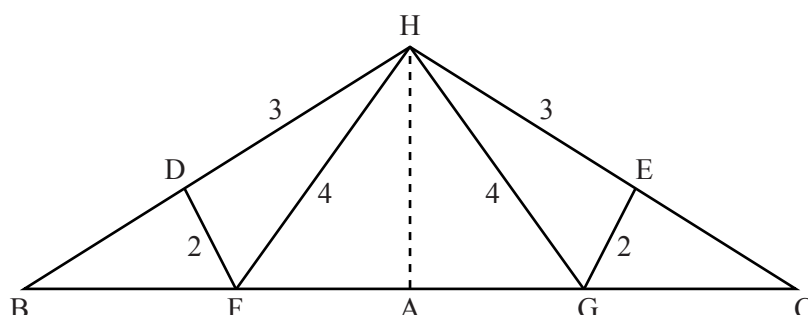


Fig. 2

- 3 A racing car team is testing a set of new tyres on a car which has a mass of 650 kg (including tyres).
- (a) The team needs to determine the coefficient of friction between the four tyres and a test track. In order to do this they place the car on a straight, level section of track with the wheels firmly locked and tow the car with a tractor and rope for a short distance at a constant velocity. A gauge indicates that the tension in the rope during the test remains constant at 7000 N.
- By modelling the situation as the whole weight of the car acting through a single tyre, calculate the coefficient of friction between the tyres and the track. [2]
- (b) The racing car with the same tyres is now driven along a straight, level section of the track until it reaches a speed of 40 m s^{-1} . At this speed the brakes are fully applied and the wheels lock, causing the car to slide in a straight line in the same direction it was originally travelling.
- Assuming that the coefficient of friction between the tyres and the track remains constant at the value found in part (a), determine the distance the car will slide along the track. [3]
- (c) In another test, the car is driven at a constant speed of 25 m s^{-1} down a straight section of the track that slopes uniformly at 10° to the horizontal. The brakes are then applied and the wheels lock, causing the car to slide in a straight line in the same direction it was originally travelling.
- Assuming that the coefficient of friction continues to have the value found in part (a), determine the distance the car will slide after the brakes are applied and the time it will take for the car to come to rest. [4]

- 4 In this question take acceleration due to gravity, g , to be 10.

Fig. 4 shows a projectile launched at an angle of α° from the edge of a cliff which is H m above sea level; the initial velocity is $v_0 \text{ m s}^{-1}$. It is assumed that the only force acting on the projectile while it is in motion is due to gravity (hence air resistance can be neglected).

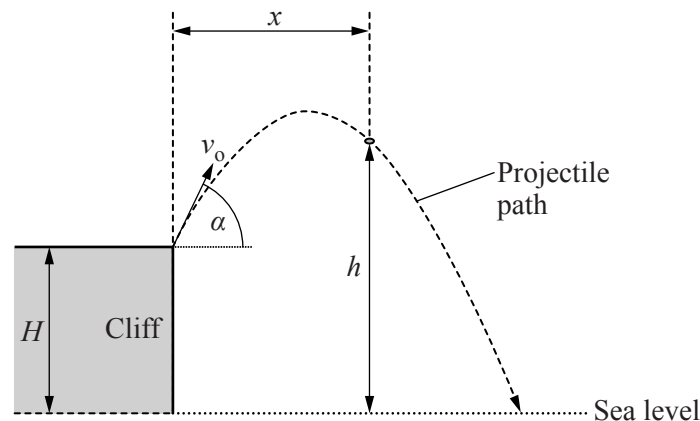


Fig. 4

- (a) Show that the height of the projectile above sea level, h m, at time t s after projection, while it is in motion, is given by

$$h = -\frac{gt^2}{2} + v_0 t \sin \alpha + H. \quad [3]$$

- (b) The relationship between the horizontal distance, x m, travelled by the projectile and the time, t s, after projection is $x = v_0 t \cos \alpha$.

Eliminate t from the expression for h given in part (a) and determine a formula in the form $h = f(x)$ that relates the height of the projectile, h m, to the horizontal distance travelled, x m, while it is in motion. [2]

- (c) The projectile is launched at an angle of 30° from the top of a 20 m high cliff and reaches sea level at a distance of 100 m from the base of the cliff.

- (i) At what velocity is the projectile launched? [3]
- (ii) What is the maximum height of the projectile above sea level during its flight? [2]
- (iii) At what time after its launch does the projectile reach sea level? [3]

- 5 A large, circular container with a curved profile, approximately that shown in Fig. 5, is being filled with liquid at a constant rate. The container is initially empty and the height of the liquid, h m, is measured each minute for three minutes. The measurements recorded are shown in Table 5.

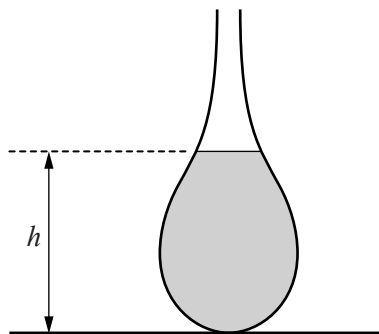


Fig. 5

Time t minutes	1	2	3
Height h metres	0.3	0.5	1.2

Table 5

The relationship between the height of the liquid, h , and time, t , is defined by the cubic equation

$$h = at^3 + bt^2 + ct + d \quad \text{for } 0 \leq t \leq 3.$$

- (a) Use the information provided to determine the values of the constants a , b , c and d . [4]
- (b) Explain why $\frac{dh}{dt}$ has a minimum value when the diameter of the container is a maximum. [1]
- (c) By finding the time at which $\frac{dh}{dt}$ is a minimum, calculate the height of the container where the diameter is a maximum. [3]
- (d) The average value, $\bar{h}$, of the function $h = f(t)$ in the interval $t_1 \leq t \leq t_2$ is given by

$$\bar{h} = \frac{\int_{t_1}^{t_2} f(t) dt}{t_2 - t_1}.$$

Use this definition to calculate the average height of the liquid during the three-minute interval. [2]

- 6 For this question you are given that the volume, V , created by rotating the region of the plane shaded in Fig. 6a, through 360° about the y -axis is given by

$$V = 2\pi \int_0^b xf(x) dx$$

where $f(x)$ is the function that describes the perimeter of the shaded region from A to B.

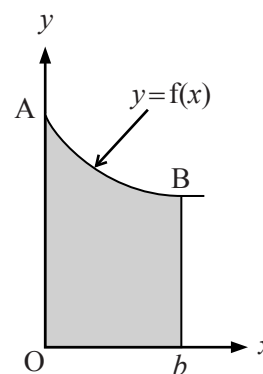


Fig. 6a

A small, circular paperweight has a cross section as shown in Fig. 6b and is symmetrical about its centre line. The profile of the cross section has the equation

$$y = h \cos^2 \frac{x\pi}{w}$$

where $h = 40$ mm and $w = 80$ mm.

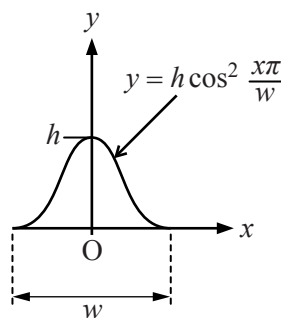


Fig. 6b

- (a) Show that the volume of the paperweight, $V \text{ mm}^3$, is given by

$$V = 80\pi \int_0^{40} x \cos^2 \frac{\pi x}{80} dx. \quad [1]$$

- (b) Calculate the volume of the paperweight.

You should use the identity $\cos^2 A = \frac{1}{2}(\cos 2A + 1)$. [6]

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