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Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate in Mathematics for Engineering **H860/01**

Paper 1

Mark Scheme for June 2010

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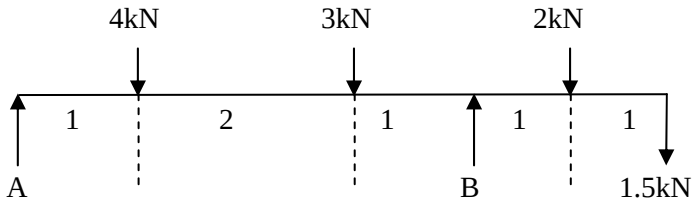
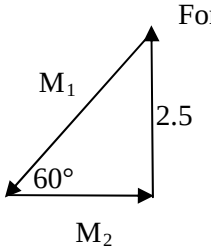
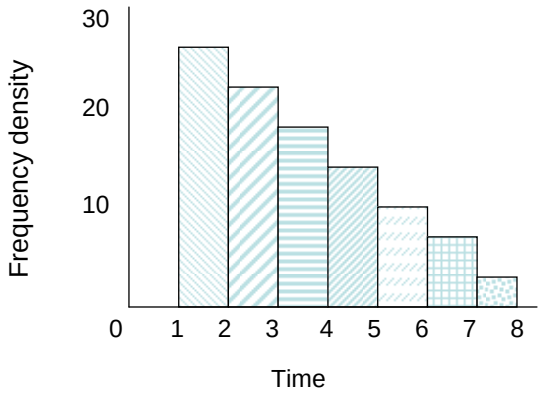
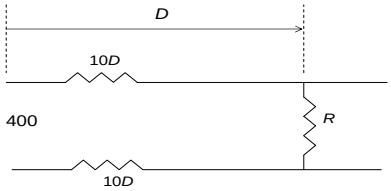
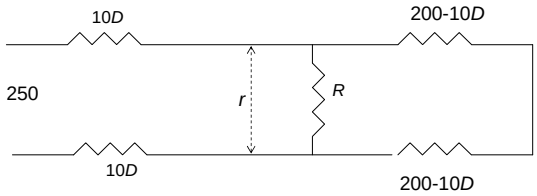
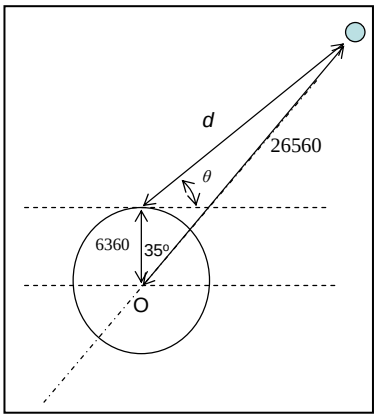
Question			Answer	Marks
1	(a)		Assuming each section is two units in length, and that the loads are equally distributed downwards, the equivalent loads are:  Taking moments about A $4\times1 + 3\times3 - B\times4 + 2\times5 + 1.5\times6 = 0$ $B = (4 + 9 + 10 + 9)/4 = 32/4 = 8$ kN Resolving forces vertically $A = 4 + 3 + 2 + 1.5 - 8 = 2.5$ kN Alternatively taking moments about B $-A\times4 + 4\times3 + 3\times1 - 2\times1 - 1.5\times2 = 0$ $A = (12 + 3 - 2 - 3)/4 = 10/4 = 2.5$kN	1 1 1 [3]
	(b)		Forces about support A  Force in $M_1 = 2.5/(\sin(60)) = 2.886...$ kN Force in $M_2 = 2.5\sin(30)/\sin(60) = 1.443...$kN (Alternatively force in $M_2 = M_1\cos(60) = 2.886\cos(60) = 1.443$ kN)	1 2 [3]
			Total	**[6]**

Question			Answer	Marks
2	(a)			2 [2]
	(b)	(i)	$P(4 \le t \le 8) = \int_4^8 \frac{1}{161}(50-6t)\mathrm{d}t$ $= \frac{1}{161}\left[50t - 3t^2\right]_4^8$ $= \frac{1}{161}\{(400-192)-(200-48)\}$ $= \frac{56}{161} = 0.3478$	1 1 1 1 [4]
		(ii)	$\frac{1}{161}\int_1^m (50-6x)\mathrm{d}t = 0.5$ where m is the median $\frac{1}{161}\left[50t - 3t^2\right]_1^m = 0.5$ $\frac{1}{161}\{(50m-3m^2)-(50-3)\} = 0.5$ $-3m^2 + 50m - 127.5 = 0$ $m = \frac{-50 \pm \sqrt{50^2 - 4\times(-3)\times(127.5)}}{2\times(-3)}$ $m = \frac{-50 \pm 31.1448}{-6}$ Use only the root $m = \frac{-50+31.1448}{-6} = 3.1425$	1 1 1 1 1 [5]
2	(c)		From the table there are 34 observations above 4 minutes. Since there are 100 observations in all, the proportion is therefore 0.34 which agrees very well with result **(b) (i)**. From the table the median is 2/18 =1/9 above 3 i.e. 3.1111. This agrees very well with result **(b) (ii)**. Based on these statistics, the given density function is a good approximation to the numerical data given in the table.	1 1 [2]
			Total	**[13]**

Question			Answer	Marks
3	(a)		The situation can be modelled as follows:  $400 = 10D + R + 10D$ $400 = 20D + R$	1 **[1]**
	(b)		 $\frac{1}{r} = \frac{1}{R} + \frac{1}{2\times(200-10D)}$ $\frac{1}{r} = \frac{2(200-10D)+R}{2R(200-10D)}$; $r = \frac{2R(200-10D)}{2(200-10D)+R}$ $250 = \frac{2R(200-10D)}{2(200-10D)+R} + 2\times 10D$ $250 = \frac{R(400-20D)}{(400-20D)+R} + 20D$	1 1 1 1 **[4]**
	(c)		Substitute $R = 400 - 20D$ $250 = \frac{(400-20D)^2}{2(400-20D)} + 20D$ $250 = \frac{(400-20D)}{2} + 20D \Rightarrow D = 5$ $R = 400 - 20 \times 5 = 300$	1 1 1 1 **[4]**
			Total	**[9]**

Question	Answer	Marks
4 (a)	The required angle is θ 	Allow rounding to 4 sf at each step
	$d^2 = 26560^2 + 6360^2 - 2\times 26560\times 6360\times\cos(35)$	2
	$d = 21660$	
	$26560/\sin(\theta + 90) = 21660/\sin(35)$	
	$\sin(\theta + 90) = 26560\times\sin(35)/21660 = 0.7033$	
	$\theta+90 = \sin^{-1}(0.7033) = 135.31$; $\theta = 45.31°$	2 **[4]**
(b)	Distance between receiver and satellite is $\sqrt{(10700-6050)^2+(15350+1100)^2+(1870-1620)^2}$	
	= 17096.42 km	
	time = $17096.42/(3\times 10^5)$ = 56.988 ms (57ms)	2 **[2]**
(c)	The required equation is: $Ax + By + Cz + D = 0$	
	$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} 1 & 21.7 & 13.2 \\ 1 & 0 & 13.3 \\ 1 & -21.7 & -13.2 \end{vmatrix} = 577.22$	2
	$B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix} = \begin{vmatrix} 7.6 & 1 & 13.2 \\ -23.0 & 1 & 13.3 \\ -7.6 & 1 & -13.2 \end{vmatrix} = -809.36$	2
	$C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 7.6 & 21.7 & 1 \\ -23.0 & 0 & 1 \\ -7.6 & -21.7 & 1 \end{vmatrix} = 998.20$	2 **[6]**
	$577.22x - 809.36y + 998.20z = 0$	
	OR other solutions e.g. $\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -23.0 & 0 & 13.3 \\ -7.6 & -21.7 & -13.2 \end{vmatrix}$	
	Giving $288.61x - 404.68y + 499.1z = 0$	
	Total	**[12]**

Question			Answer	Marks
5	(a)		$mean = \frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} \sin \omega t \, dt$	1
			$= \frac{\omega}{\pi}\left[\frac{-\cos \omega t}{\omega}\right]_0^{\frac{\pi}{\omega}}$	1
			$= \frac{1}{\pi}(1+1) = \frac{2}{\pi}$	1 [3]
	(b)		$(rms)^2 = \frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} (\sin \omega t)^2 dt$	1
			$= \frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} \frac{1}{2}(1-\cos 2\omega t) dt$	1
			$= \frac{\omega}{2\pi}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{\frac{\pi}{\omega}}$	
			$= \frac{\omega}{2\pi}\{(\frac{\pi}{\omega} - 0) - (0-0)\} = \frac{1}{2}$	1
			$rms = \frac{1}{\sqrt{2}}$	1 [4]
			Total	[7]

Question			Answer	Marks
6	(a)	(i)	$\ln a - \ln b$ Let $p = \ln a$ and $q = \ln b$ $a = e^p$ *and* $b = e^q$ $a/b = e^p e^{-q} = e^{p-q}$ $\ln a/b = \ln e^{p-q} = p - q = \ln a - \ln b$	1 1 [2]
		(ii)	$\log_{10} a = k \ln a$	
			$\log_{10} 10 = k \ln 10 = 1$	1
			$k = 1/\ln 10$	1
			$\log_{10} a = (1/\ln 10) \ln a = \frac{\ln a}{\ln 10}$	1 [3]
	(b)		$SNR = 10\log_{10}(\frac{\sigma_x^2 + \mu_x^2}{\sigma_v^2 + \mu_v^2}) =$	
			$10\log_{10}(\frac{\sigma_x^2}{\sigma_v^2}) = 10\log_{10}(\sigma_x^2) - 10\log_{10}(\sigma_v^2)$	1
			$20\log_{10}(\sigma_x) - 20\log_{10}(\sigma_v) =$	
			$\frac{20}{\ln 10}(\ln \sigma_x - \ln \sigma_v)$	1 [2]
			Total	[7]

Question	Answer	Marks
7	$x = \mathrm{e}^{-\frac{t}{2}}(A + Bt)$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-\frac{t}{2}}(B) - \frac{\mathrm{e}^{-\frac{t}{2}}}{2}(A + Bt)$	1
	$= \mathrm{e}^{-\frac{t}{2}}(B - \frac{A}{2} - \frac{Bt}{2})$	1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = e^{-\frac{t}{2}}(-\frac{B}{2}) - \frac{e^{-\frac{t}{2}}}{2}(B - \frac{A}{2} - \frac{Bt}{2})$	1
	$= e^{-\frac{t}{2}}(-B + \frac{A}{4} + \frac{Bt}{4})$	1
	$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = e^{-\frac{t}{2}}(-4B + A + Bt)$	
	$4\frac{\mathrm{d}x}{\mathrm{d}t} = e^{-\frac{t}{2}}(4B - 2A - 2Bt)$	
	$x = e^{-\frac{t}{2}}(A + Bt)$	1
	$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + x$	
	$= e^{-\frac{t}{2}}(-4B + 4B + A - 2A + A + Bt - 2Bt + Bt) = 0$	1 **[6]**
	Total	**[6]**

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