OCR
RECOGNISING ACHIEVEMENT

LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

Paper 1

H860/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 2 hours

H86001

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 Consider the framed structure shown in Fig. 1 which consists of eleven light, rigid structural members. The structure consists of five equilateral triangular sections supported at points A and B. The whole structure is arranged in the vertical plane and each joint is freely pin-jointed. Loads of 4 kN, 3 kN, 2 kN and 1.5 kN are applied at points W, X, Y and Z respectively.

(a) Determine the reaction forces of the supports on the framework at points A and B. **[3]**

(b) Determine the forces in the member M_1 and the member M_2. **[3]**

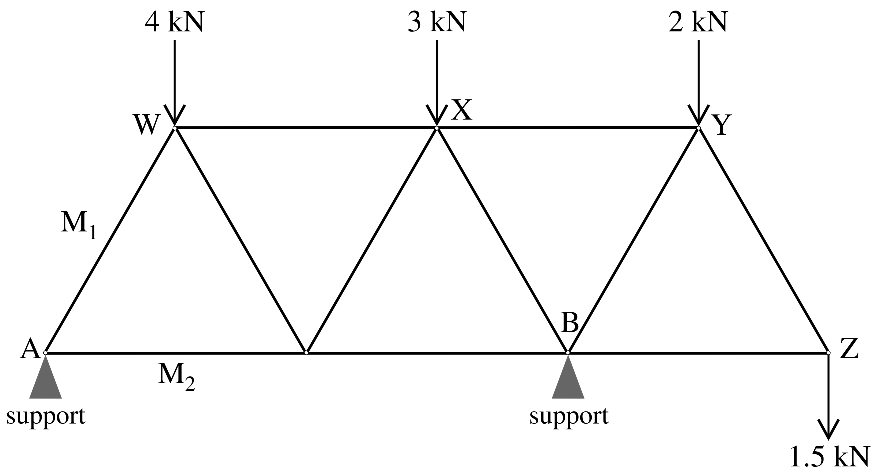

Fig. 1

2 In a certain manufacturing process the times, in minutes, for Operation A to be completed by a machine operator during a morning shift period were observed and recorded. The results are summarised in Table 2.

Operation time (minutes)	Number of observations
> 1 and $\leqslant 2$	26
> 2 and $\leqslant 3$	22
> 3 and $\leqslant 4$	18
> 4 and $\leqslant 5$	14
> 5 and $\leqslant 6$	10
> 6 and $\leqslant 7$	7
> 7 and $\leqslant 8$	3

Table 2

(a) Draw a histogram of the observations summarised in the table. **[2]**

It has been suggested that the distribution of the actual times taken for Operation A over a long period can be approximated by the probability density function

$$f(t) = \frac{1}{161}(50 - 6t) \qquad \text{for } 1 < t \leqslant 8,$$

where t is the time in minutes.

It is assumed that Operation A will take more than 1 minute and will take no more than 8 minutes.

(b) **(i)** Use this probability density function to calculate the proportion of the times taken for Operation A that are predicted to be greater than 4 minutes. **[4]**

(ii) Calculate the median of this probability density function. **[5]**

(c) Based on your answers to part **(b)**, state, with reasons, whether the suggested probability density function provides a good approximation to the data given in Table 2. **[2]**

3 For this question you may assume the following laws regarding the total resistance, R, for an electrical circuit.

For resistors connected in series, $R = R_1 + R_2 + \ldots + R_n$.

For resistors connected in parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$.

An electrical cable of length 20 km contains two copper wires each with a resistance of $10\,\Omega\,\text{km}^{-1}$. The cable is known to have slight damage at a position D km from one end which is causing a small current leakage. At the point of damage the two copper wires are effectively joined with a resistance of $R\,\Omega$, as shown in Fig. 3.

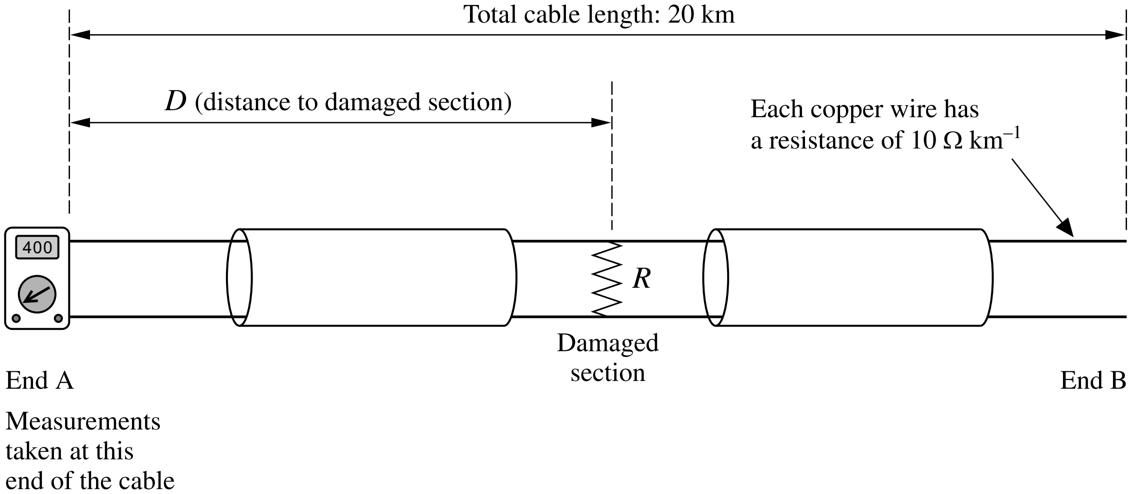

Fig. 3

In order to determine the position of the damaged section, engineers have observed that the DC resistance across the wires at end A is $400\,\Omega$ when the wires at end B are open-circuited. When the wires at end B are short-circuited, the DC resistance across the wires at end A is $250\,\Omega$.

(a) When the two wires at end B of the cable are open-circuited, the resistance measured at end A corresponds to three resistors in series consisting of the two sections of cable up to the point of damage and the resistance $R\,\Omega$.

Write down an equation which relates the measured resistance, $400\,\Omega$, to resistance $R\,\Omega$ and distance D km. **[1]**

(b) When the two wires at end B of the cable are short-circuited, the total resistance measured at end A involves the resistance of the whole length of the cable and the resistance $R\,\Omega$ in a combined parallel and series arrangement. In this case show that

$$250 = \frac{R(400 - 20D)}{400 - 20D + R} + 20D.$$ **[4]**

(c) Determine the values of R and D. **[4]**

4 In this question Ox, Oy and Oz are three mutually perpendicular axes where O is the centre of the Earth, as shown in Fig. 4a. The units of these axes are km, and the Earth is assumed to be a perfect sphere of radius 6360 km.

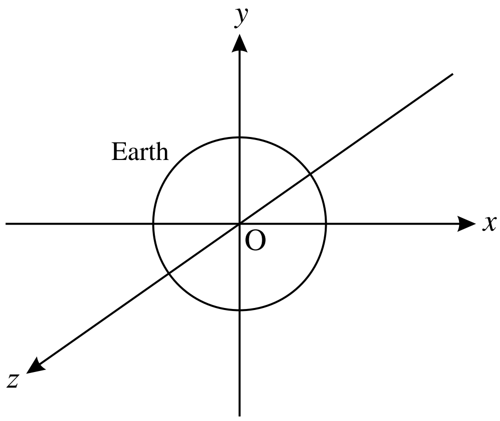

Fig. 4a

Global positioning satellites orbit the Earth at a constant altitude of 20 200 km from the Earth's surface. Fig. 4b shows a satellite at the highest position within its orbital plane, which is inclined at an angle of 55° to the plane of the equator.

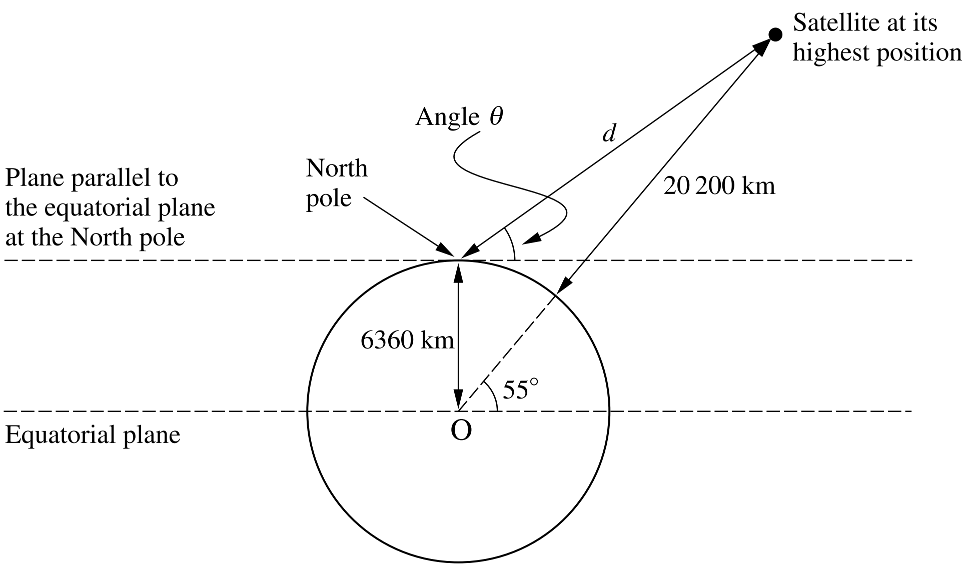

Fig. 4b

(a) Calculate the distance d and the angle θ indicated in Fig. 4b. **[4]**

(b) A satellite at (10 700, 15 350, 1870) transmits a signal. Calculate the time taken by the signal to reach a receiver at (6050, –1100, 1620), assuming that signal propagation speed is 3×10^5 km s^{-1}. **[2]**

(c) Three satellites have positions

(7600, 21 700, 13 200), (–23 000, 0, 13 300), (–7600, –21 700, –13 200).

Determine the equation of the plane in which these satellites lie, given that this plane passes through the origin. Express your answer in the form $Ax + By + Cz = 0$, where A, B, and C are constants. **[6]**

5 A continuous function $\mathrm{f}(t)$ is defined on a time interval $a \leqslant t \leqslant b$. In this question you may assume that

the mean value of $\mathrm{f}(t)$ is $\dfrac{1}{b-a}\displaystyle\int_a^b \mathrm{f}(t)\,\mathrm{d}t$,

the root mean square value (r.m.s.) of $\mathrm{f}(t)$ is $\sqrt{\dfrac{1}{b-a}\displaystyle\int_a^b (\mathrm{f}(t))^2\,\mathrm{d}t}$.

The output of a particular electrical device is an alternating current which may be represented by the function $\mathrm{f}(t) = \sin(\omega t)$, where ω is the fundamental frequency in rad s^{-1}.

(a) Determine the mean value of $\mathrm{f}(t)$ over the time interval $0 \leqslant t \leqslant \dfrac{\pi}{\omega}$. **[3]**

(b) Determine the root mean square value of $\mathrm{f}(t)$ over the time interval $0 \leqslant t \leqslant \dfrac{\pi}{\omega}$. **[4]**

6 **(a)** Starting with the definition

$$y = a^x \Leftrightarrow \log_a y = x, \text{ where } a > 1,$$

prove that

(i) $\ln a - \ln b = \ln\left(\dfrac{a}{b}\right)$, **[2]**

(ii) $\log_{10} a = \dfrac{\ln a}{\ln 10}$. **[3]**

(b) The power of a digitally transmitted signal, P_x, is given by

$$P_x = \sigma_x^2 + \mu_x^2$$

where μ_x is the mean value of the signal,
σ_x^2 is the variance of the signal.

The signal is contaminated by noise. The power of the noise is similarly given by $P_v = \sigma_v^2 + \mu_v^2$.

The signal-to-noise ratio (SNR), in decibels, of the transmitted signal is defined as

$$SNR = 10\log_{10}\left(\frac{P_x}{P_v}\right).$$

Prove that, if the mean values of both the noise and the signal are zero, then

$$SNR = \frac{20}{\ln 10}(\ln \sigma_x - \ln \sigma_v).$$ **[2]**

7 The behaviour of a simple car suspension unit can be modelled by the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0,$$

where t represents time,
x represents a displacement from the equilibrium position,
m is the mass supported by the suspension unit,
c is the damping coefficient,
k is the spring stiffness.

Given that $m = 4$, $c = 4$ and $k = 1$, verify that

$$x = e^{-\frac{1}{2}t}(A + Bt)$$

satisfies the differential equation, where A and B are constants. **[6]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.