Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/01 Paper 1

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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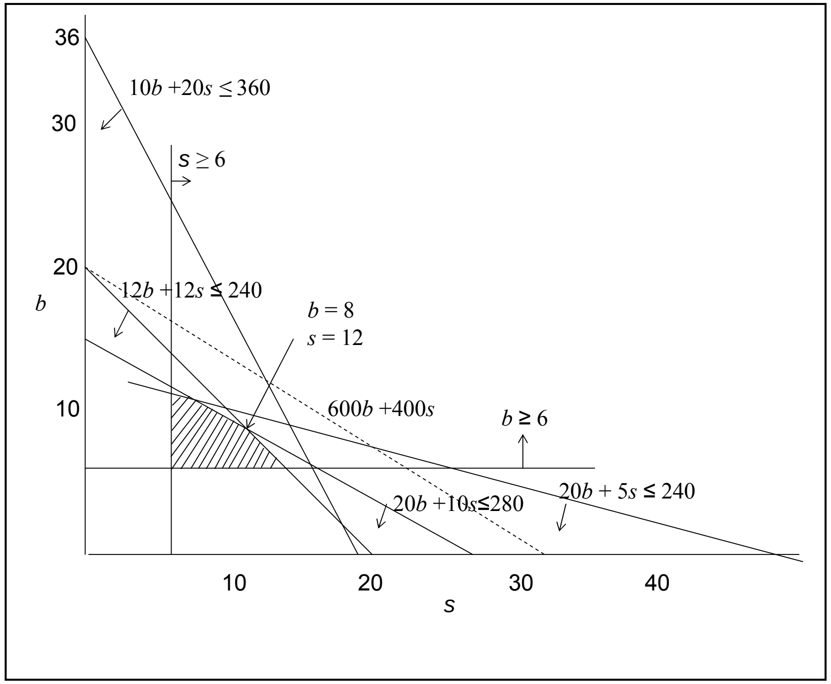
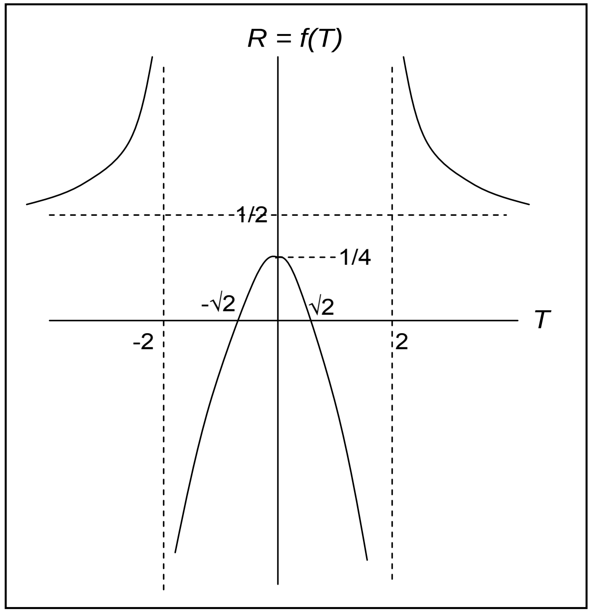
1 a i	Total hours in workshop 1 must be ≤ 480 Total hours in workshop 1 $=10b + 20s + 15a$ Therefore $10b + 20s + 15a \leq 480$	1 **1**	Must provide adequate explanation for one mark.
1 a ii	$12b + 12s + 10a \leq 320$ $20b + 10s + 10a \leq 360$ $20b + 5s + 15a \leq 360$ $b, s, a \geq 6$ Maximise $600b + 400s + 450a$	1 1 1 **3**	1 mark for three $\leq$ constraints 1 mark for ≥ 6 constraints 1 mark for objective function
1 b i	$10b + 20s \leq 480 - 15 \times 8$ (360) $12b + 12s \leq 320 - 10 \times 8$ (240) $20b + 10s \leq 360 - 10 \times 8$ (280) $20b + 5s \leq 360 - 15 \times 8$ (240) $b, s \geq 6$ Maximise $600b + 400s + 3600$	1 1 **2**	1 mark for 260, 240, 280 and 240 seen Accept $600b + 400s$
1 b ii	Constraints on graph	3 **3**	1 mark for each two $\leq$ constraints 1 mark for ≥ 6 constraints
1 b iii	Feasible region indicated	1 **1**	Accept ECF from b ii

1 b iv	$b = 8$, s =12 from graph 	1 **1**	Must indicate objective line but accept ECF from b ii and b iii
2 a	$p = 0.2$, $q = 0.8$ P(0) = 10!/(0! × (10 – 0)!) × 0.2^0 × $0.8^{10\text{-}0}$ = 0.1074 P(1) = 10!/(1! × (10 – 1)!) × 0.2^1 × $0.8^{10\text{-}1}$ = 0.2684 P(2 or more) = 1 – (0.1074 + 0.2684) = 0.6242	 1 1 1 **3**	1 mark for use of $P(X) = \frac{N!}{X!(N-X)!}0.2^X 0.8^{N-X}$ 1 mark for correct answer to one or more of P(0) and P(1) 1 mark for final result with ECF

2 b i	Mean = Np = 20 Variance = Npq = 16	1 **1**	1 mark for 20 and 16 seen
2 b ii	N(Np, Npq) = N(20, 16) P($17.5 \le x \le 24.5$) P($(17.5 - 20)/4 \le z \le (24.5 - 20)/4$) P($-0.625 \le z \le 1.125$) = 0.234 + 0.3696 = 0.6036	1 1 1 **3**	Allow ECF from b i Allow P($(18 - 20)/4 \le z \le (24 - 20)/4$) With final answer 0.1915 + 0.3413 = 0.5328
2 c	Poisson distribution $\lambda = 5$ P(x) = $5^x e^{-5}/x!$, P(0) = 0.0067, P(1) = 0.0337, P(2) = 0.0842 P($x \le 2$) = P(0) + P(1) + P(2) = 0.1246	1 1 1 **3**	1 mark for use of $\dfrac{\lambda^x e^{-\lambda}}{x!}$ 1 mark for at least two correct results Accept 0.1247 directly from table Accept ECF from previous step

3 a	$h = (e^{3/2} + e^{-3/2}) = 4.7048$	1 **1**	1 mark for 4.4817 + 0.2231 seen
3 b	Total Area $= 2\int_0^3 (e^{x/2} + e^{-x/2})\mathrm{d}x$ $= 2\left[2e^{x/2} - 2e^{-x/2}\right]_0^3$ $2\left[(2e^{3/2} - 2e^{-3/2}) - (2-2)\right]$ $4(e^{3/2} - e^{-3/2}) = 17.0342$	1 1 1 1 **4**	Hyperbolic functions do not appear in the specification, however, if candidates recognize that $y = (e^{x/2} + e^{-x/2}) =$ 2 cosh x then use the standard integral (2 sinh x) and arrive at the correct answer, 4 marks should be awarded. Allow 1 mark for reasonable approximation using straight lines for roof or trapezoidal/Simpson's rule etc.
3 c i	$y = (e^{x/2} + e^{-x/2})$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(e^{x/2} - e^{-x/2})$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{1}{4}(e^{x} + e^{-x} - 2)$ $1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{1}{4}(e^{x} + e^{-x} + 2) = \frac{1}{4}(e^{x/2} + e^{-x/2})^2$ $\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} = \frac{1}{2}(e^{x/2} + e^{-x/2})$	1 1 1 **3**	Correct solutions using hyperbolic functions should also gain full marks

3 c ii	$\frac{1}{2}\int_0^3 (e^{x/2}+e^{-x/2})\mathrm{d}x = \frac{17.0342}{4} = 4.2585$	1 **1**	Allow FT from part b
4 a	$\mathrm{f}(T) = A + \frac{B}{(T-2)} + \frac{C}{(T+2)}$ $A=\frac{1}{2}\quad B=\frac{1}{4}\quad C=-\frac{1}{4}$ $\frac{1}{2}+\frac{1}{4(T-2)}-\frac{1}{4(T+2)}$	2 **2**	OE
4 b i	$\frac{dR}{dT} = -\frac{1}{4(T-2)^2}+\frac{1}{4(T+2)^2} = 0$ when $T=0$, $R=1/4$	2 **2**	Allow ECF from 4 a
4 b ii	Poles at $T=\pm 2$ Zeros at $T=\pm\sqrt{2}$	2 **2**	

4 b iii		1 1 **2**	1 mark for general shape of upper part of graph 1 mark for general shape of lower part of graph
4 c	$T^2 - 2 = 2(T^2 - 4)R$ $T^2 - 2 = 2T^2R - 8R$ $T^2(1-2R) = 2 - 8R$ $T = \pm\sqrt{\dfrac{2-8R}{1-2R}}$ $T = \pm\sqrt{\dfrac{2-8R}{1-2R}}$ T is real when R >1/2 or R ≤ 1/4	1 2 **3**	

5 a i	$\overrightarrow{AB} = b - a = (0,0,12) - (6,4,9) = (-6,-4,3)$ $\overrightarrow{AD} = d - a = (4,14,9) - (6,4,9) = (-2,10,0)$	1 **1**	Also accept $\overrightarrow{AB} = -6i - 4j + 3k$ $\overrightarrow{AD} = -2i + 10j$ Also accept (6, 4, –3) and (2, –10, 0)
5 a ii	$\cos A = \frac{AB.AD}{\lvert AB\rvert \lvert AD\rvert} = \frac{12-40}{\sqrt{6^2+4^2+3^2}\sqrt{2^2+10^2}} = -0.3515$ $BAD = 110.5816^\circ$	1 1 **2**	1 mark for any reasonable attempt using 2-D geometry.
5 b	$ax + by + cz = d$ $a = \begin{vmatrix} 1 & 4 & 9 \\ 1 & 0 & 12 \\ 1 & 14 & 9 \end{vmatrix} = -30$ $b = \begin{vmatrix} 6 & 1 & 9 \\ 0 & 1 & 12 \\ 4 & 1 & 9 \end{vmatrix} = -6$ $c = \begin{vmatrix} 6 & 4 & 1 \\ 0 & 0 & 1 \\ 4 & 14 & 1 \end{vmatrix} = -68$ $d = \begin{vmatrix} 6 & 4 & 9 \\ 0 & 0 & 12 \\ 4 & 14 & 9 \end{vmatrix} = -816$ $-30x - 6y - 68z = -816$ $15x + 3y + 34z = 408$	1 1 1 1 1 **5**	Accept alternative solution using vector product $\begin{vmatrix} i & j & k \\ -6 & -4 & 3 \\ -2 & 10 & 0 \end{vmatrix} = -30i - 6j - 68k$ $-30x - 6y - 68z = d$ From point B $d = -68 \times 12 = -816$ $-30x - 6y - 68z = -816$ $15x + 3y + 34z = 408$

5 c	$\cos(90) = \dfrac{\overrightarrow{DC}\,\overrightarrow{DA}}{\lvert\overrightarrow{DC}\rvert \times \lvert\overrightarrow{DA}\rvert} = 0$ $\overrightarrow{DC} = (x-4, y-14, z-9)$ $\overrightarrow{DA} = (2, -10, 0)$ $\overrightarrow{DC}\,\overrightarrow{DA} = (2(x-4) - 10(y-14)) = 0$ $2x - 10y + 132 = 0$ when $x = 2$ $y = 13.6$ From $15x + 3y + 34z = 408$ $z = (408 - 30 - 40.8)/34 = 9.92$ to 3 s.f.	1 1 1 1 **4**	 Allow ECF
6 a	Output from A = $\cos(\omega_c t)\cos(\omega_o t)$ Output from B = $\sin(\omega_c t)\sin(\omega_o t)$ $\cos(\omega_c t)\cos(\omega_o t) + \sin(\omega_c t)\sin(\omega_o t) = \cos((\omega_c - \omega_o)t)$ (Using $\cos(A - B) = \cos A \cos B + \sin A \sin B$ from list MF1)	1 1 **2**	
6 b	Output from A = $\cos(\omega_c t)\cos(\omega_o t)$ $= \cos(((\omega_c + \omega_o) + (\omega_c - \omega_o))t/2)\cos(((\omega_c + \omega_o) - (\omega_c - \omega_o))t/2)$ $= \frac{1}{2}\cos(\omega_c + \omega_o)t + \frac{1}{2}\cos(\omega_c - \omega_o)t$ Output from B = $\sin(\omega_c t)\sin(\omega_o t)$ $\sin(((\omega_c + \omega_o) + (\omega_c - \omega_o))t/2)\sin(((\omega_c + \omega_o) - (\omega_c - \omega_o))t/2)$ $= -\frac{1}{2}\cos(\omega_c + \omega_o)t + \frac{1}{2}\cos(\omega_c - \omega_o)t$	1 1 1 **3**	

6 c	Output from B = $\sin(\omega_c t + \varphi)\ \sin\omega_o t$ $= (\sin\omega_c t\cos\varphi + \cos\omega_c t\sin\varphi)\sin\omega_o t$ $= \sin\omega_c t\sin\omega_o t\cos\varphi + \cos\omega_c t\sin\omega_o t\sin\varphi$	1 1 **2**	

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