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RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/02 Paper 2

Mark Scheme for June 2012

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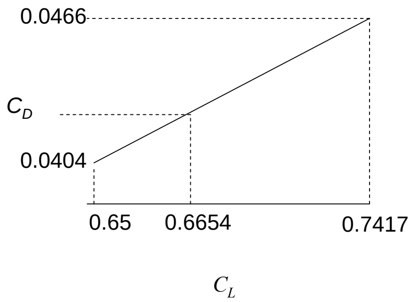
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Question			Answer	Marks	Guidance
1	(a)	(i)	$L = 40\,000g = 392$ kN	1 **[1]**	For level flight $L = W = mg$
		(ii)	$D = \frac{1}{2}C_D\rho v^2 S$ $D = \frac{1}{2}0.035 \times 0.5 \times 176^2 \times 120 \approx 33$ kN	1 **[1]**	
	(b)	(i)	For constant speed $D = F$ From $D = \frac{1}{2}C_D\rho v^2 S$ $v^2 = \frac{2D}{C_D\rho S} = \frac{2 \times 25000}{0.03 \times 0.5 \times 120} = 27777.\dot{7}$ $v = 166.\dot{6}$ m s^{-1}	1 1 **[2]**	
		(ii)	For level flight $L = W$ From $L = \frac{1}{2}C_L\rho v^2 S$ $C_L = \frac{2W}{\rho v^2 S} = \frac{2 \times 40000 \times 9.8}{0.5 \times 27777.\dot{7} \times 120} = 0.4704$	1 1 **[2]**	Alternatively using $D/L = C_D/C_L$ $C_L = C_D L/D$ $= \frac{0.03 \times 40000 \times 9.8}{25000} = 0.4704$
		(iii)	The increase in thrust will mean initially that D < F. This will cause a steady <u>increase in velocity.</u> The increased velocity will cause an increased lift force which will cause the aircraft to <u>gain height</u>.	1 1 **[2]**	

Question			Answer	Marks	Guidance
2	(a)	(i)	From $L = \frac{1}{2} C_L \rho v^2 S$ $L = \frac{1}{2} 0.665 \times 0.5 \times 180^2 \times 100 = 538650 \approx 539 \text{ kN}$	1	
			$W = mg = 55000 \times 9.8 = 539 \text{ kN}$	1 **[2]**	
		(ii)	$D = \frac{1}{2} C_D \rho v^2 S$ $D = \frac{1}{2} 0.042 \times 0.5 \times 180^2 \times 100 = 34020 \approx 34 \text{ kN}$	1 **[1]**	
		(iii)	$\beta = \frac{D}{L} = \frac{34}{539} \approx 0.0631$ $f = \frac{F}{W} = \frac{60}{539} \approx 0.1113$	1 1 **[2]**	Also accept $\beta = \frac{C_D}{C_L} = \frac{0.042}{0.665} \approx 0.0632$
		(iv)	$f - \beta = \sin \gamma \;\; \gamma = \sin^{-1}(f - \beta)$ $= \sin^{-1}(0.1113 - 0.0631) \approx 2.76°$	1 1 **[2]**	Also allow $F = D + W \sin(\gamma) \Rightarrow \lambda = \sin^{-1}\left(\frac{F - D}{W}\right)$ $= \sin^{-1}\left(\frac{60000 - 34000}{539000}\right) \approx 2.76°$ Allow 1 mark only for $\cos \gamma = \frac{L}{W} \Rightarrow \lambda = \cos^{-1}\left(\frac{L}{W}\right)$ $= \cos^{-1}\left(\frac{538650}{539000}\right) \approx 2.07°$

Question			Answer	Marks	Guidance
	(b)	(i)	$C_{\text{L}} = \frac{2W}{\rho v^2 S} = \frac{2 \times 55000 \times 9.8}{0.5 \times 180^2 \times 100} \approx 0.6654$	2 **[2]**	Allow rounding to 0.67
		(ii)	Using the table and linear interpolation $\frac{0.6654 - 0.65}{C_{\text{D}} - 0.0404} = \frac{0.7417 - 0.65}{0.0466 - 0.0404}$ $C_{\text{D}} = \left(\frac{0.6654 - 0.65}{0.7417 - 0.65}\right) \times (0.0466 - 0.0404) + 0.0404 \approx 0.0414$	1 1 **[2]**	 The answer must refer to the values in the table and show that a linear relationship has been used. Award no marks for a value with no working. Award 1 mark for answer between 0.0404 and 0.0466 with some evidence of reasoning.
		(iii)	$\beta = \frac{C_{\text{D}}}{C_{\text{L}}} = \frac{0.0414}{0.6654} \approx 0.0622$ using $\frac{F}{W} - \beta = \sin\gamma$ $F = W \times (\sin\gamma + \beta)$ $= 539000 \times (\sin(4) + 0.0622) \approx 71\text{kN}$	1 1 1 **[3]**	Allow ECF from (b)(i) and (b)(ii). Allow alternative solution $F = D + W\sin\gamma$ $= 34 + 550000g\sin 4$ ≈ 71 kN

Question			Answer	Marks	Guidance
3			$\beta = \frac{4\alpha^2 + 10\alpha + 200}{10000} \times (0.09\alpha + 0.1)^{-1}$	1	
			$\frac{d\beta}{d\alpha} = -0.09(0.09\alpha + 0.1)^{-2} \times \left(\frac{4\alpha^2 + 10\alpha + 200}{10000}\right) + \left(\frac{8\alpha + 10}{10000}\right) \times (0.09\alpha + 0.1)^{-1}$	3	Award 1 mark for $8\alpha + 10$ seen Also accept for 3 marks $\frac{(900\alpha + 1000)(8\alpha + 10) - (4\alpha^2 + 10\alpha + 10)900}{(900\alpha + 1000)^2}$ OE
			For minimum α, $\frac{d\beta}{d\alpha} = 0$		
			$-0.09 \times (4\alpha^2 + 10\alpha + 200) + (8\alpha + 10) \times (0.09\alpha + 0.1) = 0$		
			$(-0.36 + 0.72)\alpha^2 + (-0.9 + 0.8 + 0.9)\alpha + (-18 + 1) = 0$		
			$0.36\alpha^2 + 0.8\alpha - 17 = 0$	1	
			$\alpha = \frac{-0.8 \pm \sqrt{0.8^2 + 4 \times 0.36 \times 17}}{2 \times 0.36}$	1	
			$= \frac{-0.8 \pm 5.012}{0.72}$		
			α must be +ve so $\alpha = 5.85$	1 **[7]**	

Question			Answer	Marks	Guidance
4	(a)	(i)	$v = v_S$ when $L = W = 45000 \times 9.8 = 441000$ $v_S{}^2 = \dfrac{2W}{\rho S C_L} = \dfrac{2 \times 441000}{1.22 \times 120 \times 1.2} = 5020.4918$ $v_S = \sqrt{5020.4918} = 70.8554$ m s^{-1}	1 1 **[2]**	
		(ii)	$a = \dfrac{150000}{45000}$ $t = \dfrac{v_S}{a} = \dfrac{70.8554 \times 45000}{150000} = 21.2566$ s	1 1 **[2]**	
4	(b)	(i)	$m\dfrac{dv}{dt} = F - D - D_R$ $m\dfrac{dv}{dt} = F - \dfrac{1}{2}\rho v^2 C_D S - \mu_R (W - \dfrac{1}{2} C_L \rho v^2 S)$ $m\dfrac{dv}{dt} = F - (\dfrac{1}{2}\rho v^2 S(C_D - \mu_R C_L) + \mu_R W)$	2 **[2]**	

Question			Answer	Marks	Guidance
		(ii)	$\frac{1}{v_S}\int_0^{v_S} F-(\frac{1}{2}\rho v^2 S(C_D-\mu_R C_L)+\mu_R W)\,dv$ $\frac{1}{v_S}\left[Fv-(\frac{1}{2}\rho\frac{v^3}{3}S(C_D-\mu_R C_L)+\mu_R Wv)\right]_0^{v_S}$ $=F-(\frac{1}{2}\rho\frac{v_S^2}{3}S(C_D-\mu_R C_L)+\mu_R W)$ $=150000-(\frac{1}{2}\times 1.22\times 120\times\frac{5020.4918}{3}\times(0.07-0.025\times 1.2)+0.025\times 441000)$ = 134 075 N	 1 1 1 **[3]**	Accept numerical equivalent $\frac{1}{70.8554}\int_0^{70.8554} 138975-2.928v^2\,dv$ $\frac{1}{70.8554}\left[138975v-\frac{2.928v^3}{3}\right]_0^{70.8554}$ $=138975-\frac{2.928(70.8554)^2}{3}$ = 134 075 N Accept answer rounded to the nearest kN (≈ 134 kN)
		(iii)	From $F=ma$ $134075=m\frac{dv}{dt}$ $134075t=mv$ $t=\frac{mv}{134075}=\frac{45000\times 70.8554}{134075}\approx 23.78$ s	 1 1 **[2]**	 Accept rounded answer to 24 s

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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