OCR
RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/01 Paper 1

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

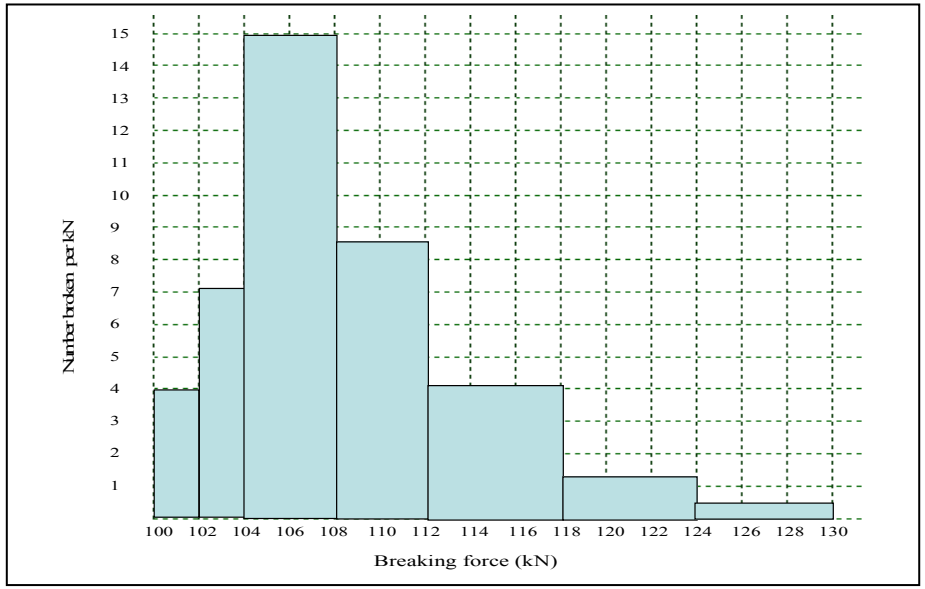
All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>1</td><td>(a)</td><td></td><td>
<table>
<tr><th>Breaking force, F kN</th><th>Mid-class value X</th><th>f</th><th>d= X-100</th><th>fd</th><th>fd^2</th></tr>
<tr><td>$0 < F \leq 100$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$100 < F \leq 102$</td><td>101</td><td>8</td><td>1</td><td>8</td><td>8</td></tr>
<tr><td>$102 < F \leq 104$</td><td>103</td><td>14</td><td>3</td><td>42</td><td>126</td></tr>
<tr><td>$104 < F \leq 108$</td><td>106</td><td>60</td><td>6</td><td>360</td><td>2160</td></tr>
<tr><td>$108 < F \leq 112$</td><td>110</td><td>34</td><td>10</td><td>340</td><td>3400</td></tr>
<tr><td>$112 < F \leq 118$</td><td>115</td><td>24</td><td>15</td><td>360</td><td>5400</td></tr>
<tr><td>$118 < F \leq 124$</td><td>121</td><td>8</td><td>21</td><td>168</td><td>3528</td></tr>
<tr><td>$124 < F \leq 130$</td><td>127</td><td>2</td><td>27</td><td>54</td><td>1458</td></tr>
<tr><td>Sum</td><td></td><td>150</td><td></td><td>1332</td><td>16080</td></tr>
<tr><td></td><td></td><td></td><td></td><td>8.88</td><td>107.2</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>3

[3]</td><td>Allow 1 mark for mid-class values
Allow 2 marks for frequencies
Allow ± 2 for all values of f

Marks for values other than X and f awarded in parts (c)(ii) and (c)(iii)</td></tr>
<tr><td>1</td><td>(b)</td><td></td><td> 
</td><td>4
[4]</td><td>1 mark for horizontal and vertical axes

1 mark for general shape

1 mark for correct widths

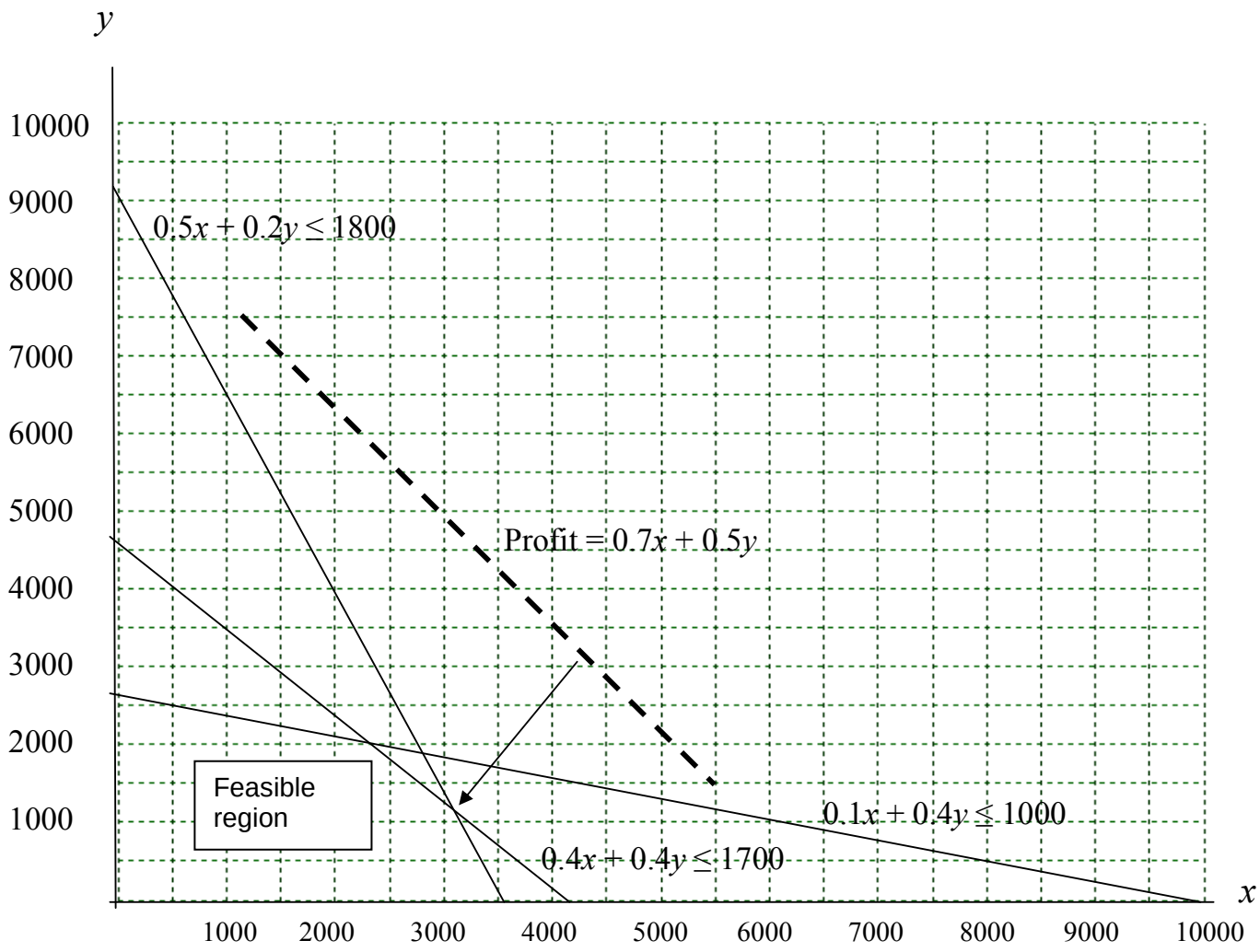
1 mark for correct heights</td></tr>
</table>

Question			Answer	Mark	Guidance
1	(c)	(i)	mean = 100 + 1332/150 = 108.88	2 [2]	1 for value 1332 (OE) from table 1 for answer with ECF Allow 1 mark for $\frac{\sum fd}{\sum f}$ OE seen
1	(c)	(ii)	std = $\sqrt{\frac{16080}{150} - 8.88^2} = 5.3241$	3 [3]	2 for values 16080 and 8.88 (OE) from table 1 for answer with ECF Allow 1 mark for $\sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$ OE seen

Question			Answer	Mark	Guidance
2	(a)		Total resistance of circuit **ai** $\frac{1}{1.2}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{3}$		
			Total resistance of circuit **aii** $\frac{1}{2.5}=\frac{1}{R_1+R_2}+\frac{1}{3}$	1	
			From **ai** $\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{1.2}-\frac{1}{3}=\frac{1}{2}$	1	
			From **aii** $\frac{1}{R_1+R_2}=\frac{1}{2.5}-\frac{1}{3}=\frac{1}{15} \Rightarrow R_1+R_2=15$ and $R_1=15-R_2$	1	or $R_2=15-R_1$
			$\frac{1}{15-R_2}+\frac{1}{R_2}=\frac{1}{2}$		
			$\frac{R_2+(15-R_2)}{(15-R_2)R_2}=\frac{15}{(15R_2-R_2^2)}=\frac{1}{2}$		
			$R_2^2-15R_2+30=0$	1	
			$R_2=\frac{15\pm\sqrt{15^2-120}}{2}$		
			$R_2=12.6235\ \Omega$	1	
			$R_1=2.3765\ \Omega$	1	
				[6]	

Question			Answer	Mark	Guidance
2	(b)	(i)	$R_{AB} = \frac{2}{3}\Omega$	1	
			$R_{BC} = \frac{24}{14} = \frac{12}{7}\Omega$	1	
			Total resistance R, $\frac{2}{3} + \frac{12}{7} = \frac{14+36}{21} = \frac{50}{21}\Omega$		
			Total current $I = \frac{V}{R} = \frac{10}{50/21} = \frac{210}{50} = \frac{21}{5}$ A	1	Allow 1 mark for Ohm's law
				[3]	
2	(b)	(ii)	Voltage drop $V_{AB} = IR_{AB} = \frac{21}{5} \times \frac{2}{3} = \frac{14}{5}$ V	1	
			Voltage drop $V_{BC} = IR_{BC} = \frac{21}{5} \times \frac{12}{7} = \frac{36}{5}$ V	1	
				[2]	
2	(b)	(iii)	$I_1 = \frac{V_{AB}}{R_1} = \frac{14}{5} \times \frac{1}{1} = \frac{14}{5} = 2 \cdot 8\text{A}$	1	
			$I_2 = I - I_1 = \frac{21}{5} - \frac{14}{5} = \frac{7}{5} = 1 \cdot 4\text{A}$	1	
			$I_3 = \frac{36}{5} \times \frac{1}{2} = \frac{18}{5} = 3.6\text{A}$		
			$I_4 = I - I_3 = \frac{21}{5} - \frac{18}{5} = \frac{3}{5} = 0.6\text{A}$	1	
				[3]	

Question			Answer	Mark	Guidance
3	(a)		Profit = $x(1.4 - 0.5(0.5 + 0.4 + 0.1) - 0.2) +$ $y(1.2 - 0.5(0.2 + 0.4 + 0.4) - 0.2) +$ $z(1.3 - 0.5(0.7 + 0.3) - 0.2) =$ $0.7x + 0.5y + 0.6z$	1 1 **[2]**	
3	(b)		$0.5x + 0.2y + 0.7z \leq 2500$ $0.4x + 0.4y + 0.3z \leq 2000$ $0.1x + 0.4y \leq 1000$	1 1 1 **[3]**	
3	(c)	(i)	Maximise $0.7x + 0.5y + 600$ subject to: $0.5x + 0.2y \leq 1800$ $0.4x + 0.4y \leq 1700$ $0.1x + 0.4y \leq 1000$ $x, y \geq 0$	1 1 1 **[3]**	Allow omission of 600 Allow one error or one omission

Question			Answer	Mark	Guidance
3	(c)	(ii)		4 [4]	1 mark for each constraint 1 mark for feasible region identified. (mark for objective function included with part **iii**)

Question			Answer	Mark	Guidance
3	(c)	(iii)	x between 3100 and 3300 y between 1000 and 1200 $x = 3167$ $y = 1083$	2 **[2]**	Accept answers read directly from graph OR solving for: $0.4x + 0.4y = 1700$ $0.5x + 0.2y = 1800$
4	(a)		$\frac{dN}{dt} = -\lambda N$ $\frac{dN}{N} = -\lambda dt$ $\int \frac{1}{N} dN = -\int \lambda dt \quad : \quad \ln(N) = -\lambda t + C$ $N = Ae^{-\lambda t}$ $N = N_0$ when $t = 0 \Rightarrow A = N_0$ $N = N_0 e^{-\lambda t}$	1 1 1 **[3]**	
4	(b)		$e^{-\lambda t} = \frac{N}{N_0}$ $-\lambda t = \ln\left(\frac{N}{N_0}\right)$ $-\lambda t_{1/2} = \ln\left(\frac{1}{2}\right)$ $t_{1/2} = -\frac{1}{\lambda}\ln\left(\frac{1}{2}\right) = \frac{\ln(2)}{\lambda}$	1 1 **[2]**	

Question			Answer	Mark	Guidance
4	(c)		$t = -\frac{\ln\left(\frac{N}{N_0}\right)}{\lambda} = \frac{\ln\left(\frac{N_0}{N}\right)}{\lambda}$	1	
			$N = 35\%\ N_0$		
			$t = \frac{\ln\left(\frac{N_0}{0.35N_0}\right)}{\lambda} = \frac{\ln\left(\frac{1}{0.35}\right)}{\lambda}$	1	
			But $\frac{1}{\lambda} = \frac{t_{1/2}}{\ln(2)} = \frac{5730}{\ln(2)}$	1	
			$t = \frac{\ln\left(\frac{1}{0.35}\right) \times 5730}{\ln(2)} \approx 8700$ years	1	
				[4]	
5	(a)		$a = ge^{-\frac{t}{2}} \sin t$		
			$\frac{da}{dt} = ge^{-\frac{t}{2}}(\cos t - \frac{\sin t}{2})$	1	Allow 1 mark for $(uv)'=uv'+u'v$ OE seen
			For maximum a		
			$\cos t = \frac{\sin t}{2}$	1	
			$\tan t = 2$		
			$t = \tan^{-1} 2 = 1.1071$	1	
			$a \approx 5.04$	1	
				[4]	

Question			Answer	Mark	Guidance
5	(b)	(i)	$I = \int e^{-\frac{t}{2}} \sin t \, dt = -e^{-\frac{t}{2}} \cos t - \frac{1}{2}\int e^{-\frac{t}{2}} \cos t \, dt$	1	Allow 1 mark for $\int uv' = uv - \int vu'$ OE seen
			$\int e^{-\frac{t}{2}} \cos t \, dt = e^{-\frac{t}{2}} \sin t + \frac{1}{2}\int e^{-\frac{t}{2}} \sin t \, dt$	1	
			$I = -e^{-\frac{t}{2}} \cos t - \frac{1}{2}\left(e^{-\frac{t}{2}} \sin t + \frac{1}{2} I \right)$	1	
			$I\left(1 + \frac{1}{4}\right) = -e^{-\frac{t}{2}}(\cos t + \frac{\sin t}{2})$	1	
			$I = -\frac{4}{5} e^{-\frac{t}{2}} \left(\cos t + \frac{\sin t}{2} \right) + C$	1	
				[5]	
5	(b)	(ii)	$v = g(-\frac{4}{5} e^{-\frac{t}{2}} \left(\cos t + \frac{\sin t}{2} \right) + C)$		
			$v_0 = 0 \Rightarrow C = \frac{4}{5}$	1	Solution must solve for C
			$v_1 = 9.8(-\frac{4}{5} e^{-\frac{1}{2}} \left(\cos 1 + \frac{\sin 1}{2} \right) + \frac{4}{5}) = 3.2701 \ \text{m s}^{-1}$	1	
				[2]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553