OCR
RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/02 Paper 2

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Question			Answer	Mark	Guidance
1	(a)		$P_p = 2rfg = 4 \times 100 \times 9.8 = 3920$ W	1 **[1]**	Accept answers between 3900 and 4000
1	(b)	(i)	$\text{mass} = \frac{\Delta\theta}{\omega} f$ $\Delta\theta = \frac{2\pi}{n} = \frac{2\pi}{18} = \frac{\pi}{9}$ $\omega = \frac{5 \times 2\pi}{60}$ rad s^{-1} $= \frac{\pi}{6}$ $\text{mass} = \frac{\pi/9}{\pi/6} \times 100 = \frac{2}{3} \times 100 \approx 67$ kg	1 1 1 **[3]**	Allow 0.3491 Allow 0.5236 Allow $\frac{0.3491}{0.5236} \times 100$ with ECF
1	(b)	(ii)	$X(\theta) = 1$ for $0 \le \theta \le \frac{5\pi}{9}$ $X(\theta) = 0$ for $\theta > \frac{5\pi}{9}$	1 **[1]**	
1	(b)	(iii)	$\tau \approx \frac{fgr}{\omega} \sum_{j=1}^{n/2} X(\frac{2\pi}{n} j) \sin(\frac{2\pi}{n} j) \Delta\theta$ $\frac{fgr\Delta\theta}{\omega} \sum_{j=1}^{5} \sin(\frac{2\pi}{18} j)$ $P = fgr\Delta\theta \sum_{j=1}^{5} \sin(\frac{2\pi}{18} j)$ $P = 100 \times 9.8 \times 2 \times \frac{2\pi}{18} (\sin(\frac{\pi}{9}) + \sin(\frac{2\pi}{9}) + \sin(\frac{3\pi}{9}) + \sin(\frac{4\pi}{9}) + \sin(\frac{5\pi}{9}))$ $P = 684.169 \times 3.8205 \approx 2614$ W	1 1 1 2 **[5]**	Allow FT from **b ii**

Question			Answer	Mark	Guidance
2	(a)		$P \approx fgr\int_0^{\pi} \mathrm{X}(\theta)\sin(\theta)\ \mathrm{d}\theta$ $\mathrm{X}(\theta) = 1$ for $0 \le \theta \le \theta_A$ $\mathrm{X}(\theta) = 0$ for $\theta > \theta_A$ $P \approx fgr\int_0^{\theta_A} \sin(\theta)\ \mathrm{d}\theta$ $P = fgr\left[-\cos(\theta)\right]_0^{\theta_A}$ $P = fgr\left(-\cos(\theta_A) - (-1)\right) = fgr\left(1 - \cos(\theta_A)\right)$	1 1 1 **[3]**	Allow 1 for $\int \sin\theta = -\cos\theta$ seen
2	(b)		$P = fgr\left\{\int_0^{\frac{\pi}{2}} \sin\theta\ \mathrm{d}\theta + \int_{\frac{\pi}{2}}^{\pi} 2(1-\frac{\theta}{\pi})\sin\theta\ \mathrm{d}\theta\right\}$ $P = fgr\left\{\left[-\cos\theta\right]_0^{\pi/2} + 2\left[-\cos\theta\right]_{\pi/2}^{\pi} - \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \theta\sin\theta\ \mathrm{d}\theta\right\}$ $\int \theta\sin\theta \mathrm{d}\theta = -\theta\cos\theta + \int\cos\theta \mathrm{d}\theta = -\theta\cos\theta + \sin\theta$ $P = fgr\left\{1 + 2 - \frac{2}{\pi}\left[-\theta\cos\theta + \sin\theta\right]_{\pi/2}^{\pi}\right\}$ $P = fgr\{3 - \frac{2}{\pi}\left((\pi + 0) - (-0 + 1)\right) = fgr(1 + \frac{2}{\pi})$ $P = 100 \times 9.8 \times 2 \times (1 + \frac{2}{\pi}) \approx 3208$ W	1 1 1 1 1 **[5]**	Solution must show two integrals with correct limits Solution must demonstrate integration by parts

Question			Answer	Mark	Guidance
2	(c)		$\eta = \dfrac{\text{Output power}}{\text{Power of decending water}} = \dfrac{fgr(1-\cos\theta_A)}{2frg} = \dfrac{(1-\cos\theta_A)}{2}$ Using $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ from the formula list provided With $A = \theta_A$ and $B = 0$ $\cos\theta_a - 1 = -2\sin^2\dfrac{\theta_A}{2}$ $\dfrac{(1-\cos\theta_A)}{2} = \sin^2\dfrac{\theta_A}{2}$	1 1 1 **[3]**	Accept use of any standard formulae eg $\cos 2x = 1 - 2\sin^2 x$
3	(a)		Power at the output of the alternator $P_{out} = VI = 14.4 \times 120 = 1728$ W $P_{in} = 4 \times 100 \times 9.8 = 3920$ W $\eta = \dfrac{1728}{3920} \approx 44\,\%$	1 1 **[2]**	Allow 1 mark for $\eta = \dfrac{P_{OUT}}{P_{IN}}$ seen Accept 0.44
3	(b)		Power from inverter = 450 W Power into inverter = 450/0.9 = 500 W Current drawn by inverter = 500/14.4 ≈ 34.72 Current available to charge battery = 120 – 34.72 ≈ 85.28 A	1 1 **[2]**	Allow 1 mark for $P = VI$ OE seen Allow ECF
3	(c)	(i)	Power out of inverter = 2000 W Power in = 2000/0.9 = 2222.22 W Current reqd. = 2000/0.9 /14.4 = 154.32 A Current from battery = 154.32 – 120 = 34.32 A	1 1 **[2]**	

Question			Answer	Mark	Guidance
3	(c)	(ii)	Time to drain battery = $T \approx 20 \times \left(\frac{C_{20}}{20I} \right)$	1	
			$= 20 \times \left(\frac{200}{20 \times 34.32} \right) \approx 4$ hours	1	Allow ECF for I
				[2]	
4	(a)	(i)	$J\alpha = \tau_T$		
			$\alpha = \frac{\tau_T}{J} = \frac{50}{2000} = 0.025$	1	
			$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	1	
			$\omega_1 = \frac{5 \times 2\pi}{60} = 0.5236 \text{ rad s}^{-1}$		
			$\omega_2 = \sqrt{\omega_1^{\,2} + 2\alpha\theta} = \sqrt{0.5236^2 + 2 \times 0.025 \times 2\pi} = 0.767$ (7.234 revs per minute)	1	
				[3]	
4	(a)	(ii)	$\theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t$	1	
			$t = \frac{2 \times 2 \times \pi}{0.5236 + 0.7670} = 9.737$ s	1	Accept answers between 9.7 and 9.8
			or		
			$\omega_2 = \omega_1 + \alpha t$		
			$t = \frac{(\omega_2 - \omega_1)}{\alpha} = \frac{(0.767 - 0.5236)}{0.025} = 9.736$ s		
				[2]	

Question			Answer	Mark	Guidance
4	(b)		$J\alpha = \tau_T = \tau_D - \tau_L$		
			$J\dfrac{d\omega}{dt} = -200(1+\omega)$	1	Assuming that $\tau_D = 0$
			$\dfrac{d\omega}{(1+\omega)} = -\dfrac{200}{2000}dt$		
			$\ln(1+\omega) = -\dfrac{t}{10} + C$	1	
			$1+\omega = Ae^{-\frac{t}{10}}$	1	
			$\omega = Ae^{-\frac{t}{10}} - 1$		
			$\omega = \omega_0$ when $t = 0 \Rightarrow A = (\omega_0 + 1)$		
			$\omega = (\omega_0 + 1)e^{-\frac{t}{10}} - 1$	1	
			When $\omega = 0 \quad (\omega_0 + 1) = e^{\frac{t}{10}}$	1	
			$\dfrac{t}{10} = \ln(\omega_0 + 1)$		
			$t = 10\ln(0.5236 + 1) = 4.2108$ s	1	
				[6]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553